Prediction of Expansive Soil Volumetric Change Behavior subject to Long-term Climate Data

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ABSTRACT
The volume change behavior of expansive soils in response to climate cycles result in shrinkage and swelling of soils that can cause severe structural damage, especially to lightweight structures. Hence, prediction of expansive soil deformation behavior considering soil-atmosphere interaction is valuable to geotechnical engineers. In this study, a finite element model that couples variably-saturated flow and stress-strain analysis is used to simulate the volume change behavior of expansive clay subject to long-term climate data. The finite element model utilizes suction stress-based effective stress to calculate the effective stress field. Moreover, the model equipped with a power law equation to incorporate the dependency of soil modulus on volumetric water content of soil. The developed numerical model was validated against a benchmark problem in which a layer of Regina expansive clay is subjected to a constant infiltration rate. The obtained results indicate the capability of the proposed model in simulation of expansive soil deformations subject to varying moisture conditions over time.

RÉSUMÉ
Le comportement de changement de volume des sols expansifs en réponse aux cycles climatiques entraîne un rétrécissement et un gonflement des sols qui peuvent causer de graves dommages structurels, en particulier aux structures légères. Par conséquent, la prédiction du comportement de déformation du sol expansif en tenant compte de l'interaction sol-atmosphère est précieuse pour les ingénieurs géotechniques. Dans cette étude, un modèle par éléments finis qui couple l'analyse de l'écoulement et de la contrainte-déformation saturés de manière variable est utilisé pour simuler le comportement de changement de volume de l'argile expansive soumise à des données climatiques à long terme. Le modèle par éléments finis utilise une contrainte efficace basée sur la contrainte d'aspiration pour calculer le champ de contrainte effective. De plus, le modèle était équipé d'une équation de loi de puissance pour incorporer la dépendance du module du sol sur la teneur en eau volumétrique du sol. Le modèle numérique développé a été validé par rapport à un problème de référence dans lequel une couche d'argile expansive de Regina est soumise à un taux d'infiltration constant. Les résultats obtenus indiquent la capacité du modèle proposé à simuler des déformations expansives du sol sujettes à des conditions d'humidité variables au fil du temps.

1 INTRODUCTION

Expansive soils experience significant volume change in response to changes in soil moisture content. Climate is a key factor influencing the soil moisture content fluctuations. Climatic drying/wetting cycles may cause expansive soil undergoing significant swelling/shrinking that may ultimately lead to considerable damage to the lightly loaded buildings, buried utilities and pavements.

Adem and Vanapalli (2015) reviewed the various methods proposed for predicting in situ volume change movement of expansive soils over time. They categorized the available methods into three groups: (i) consolidation theory-based methods (e.g. Vu and Fredlund 2004) (ii) water content-based methods (e.g. Briaud et al. 2003), and (iii) suction-based methods e.g. (Wray et al. 2005).

In general, the hydro-mechanical modeling of the expansive soil response to climate variables requires transient variably-saturated seepage analysis equipped with soil-atmospheric boundary coupled with a mechanical strain-stress model based on unsaturated soil mechanics. The dependency of mechanical properties of an expansive soil on its moisture content requires an appropriate constitutive model to simulate the volumetric deformation behavior of expansive soil.

Alonso et al. (1990) developed the Barcelona Basic Model (BBM), which correlates soil hardening with suction, to simulate the unsaturated soil stress-strain behavior. Later, Alonso et al. (1999) extended the original model for expansive soils by including a conceptual fabric representation: elastic deformations in microstructure and plastic deformations in macrostructure. They validated the extended constitutive model, known as Barcelona Expansive Model (BExM) using laboratory test data to understand the combined effect of vertical stress and soil suction on net volumetric strains. Vu and Fredlund (2004, 2006) developed an elasticity-based vertical displacement model for unsaturated expansive soils that simulates the non-linear stress-strain behavior by using the coefficient of compressibility as a function of net normal stress and soil suction. They examined the developed model using coupled and uncoupled approaches for several examples and indicated that the results from an uncoupled analysis compares well with those obtained from a coupled analysis. Ito and Azem (2014) developed a two-stage model for moisture-induced deformations in expansive soils. They included a coupled soil-atmosphere model for seepage through the soil and suction-based displacement model to capture the behavior of expansive soil using normal stress and soil suction. They validated the developed model using field monitoring data, reported by Hu et al. (2010). Adem and Vanapalli (2013) proposed a simple method for predicting the vertical movements of unsaturated, expansive soils with respect to time. This model, which is referred as modulus of elasticity-based method (MEBM), integrates Fredlund and Morgenstern (1976) constitutive equation for unsaturated soil structure along with the soil-atmosphere model VADOSE/W (Geo-Slope, 2007) to predict the variation of expansive soil movement over time. Adem and Vanapalli (2015) further extended this model for estimating the modulus for unsaturated expansive soils using a semi-empirical model originally proposed by Oh et al. (2009). They employed the MEBM for several case studies to study expansive soil response over moisture variation with time (Vanapalli and Adem 2012, Adem and Vanapalli 2013, Vanapalli and Adem 2013, Adem and Vanapalli 2015). Karunarathne et al. (2018) developed a numerical model to estimate the ground movements in expansive soils for two sites in Melbourne, Australia. They used VADOSE/W to simulate the soil moisture variation due to climate. The predicted soil moistures from the VADOSE/W model were used to predict the ground movements using FLAC3D. They used the model to variation of ground movements due to long-term climate conditions.

The majority of the previous numerical models used two independent stress state variable approach in which the soil constitutive law is defined based on net normal stress and soil suction. The main contributing material properties in the elasticity-based methods within the two-stress state framework are elasticity modulus with respect to net normal stress (E) and elasticity modulus with respect to suction stress (H).

In this study, a two-dimensional finite element model that incorporates a transient variably-saturated seepage analysis and an elastic-based effective stress-strain analysis was used to simulate the volumetric behavior of expansive clay subject to variation of soil moisture due to long-term climate data. The finite element model utilizes suction stress to calculate the effective stress field as a single stress state variable. The proposed method requires elastic modulus to model the expansive soil deformation behavior under mechanical and environmental loadings comparing to the two-stress state variable framework that requires two elastic moduli. The power law equation proposed by Lu and Kaya (2014) was employed to incorporate the dependency of soil modulus on volumetric water content of soil. The developed numerical model was used to model the benchmark problem of a partially flexible pavement placed on Regina expansive clay subjected to a constant infiltration rate for 175 days, and the results were compared with those obtained by Vu and Fredlund (2006).

2 SINGLE STRESS STATE VARIABLE RAMEWORK

The effective stress is known as a fundamental stress state variable that has been extensively used to calculate the soil strength and simulate the soil deformation behavior within the classical soil mechanics framework. Bishop (1954, 1959) extended the Terzaghi's effective stress to be used for unsaturated conditions:

$$\sigma' = (\sigma - u_a) - \chi(u_w - u_a) = \sigma_{nt} - \chi h \qquad [1]$$

where u_a is pore-air pressure, u_w is pore-water pressure, $\sigma_{nt} = \sigma - u_a$ is net total stress, and $h = u_w - u_a$ is matric suction. χ is scaling parameter depending on the soil saturation degree with the constraints of being zero when soil is completely dry and being unity when soil is fully saturated. The above single-valued effective stress equation was shown to be inadequate for describing both volume change and shear strength behavior (Houston, 2019; Zhang and Lu, 2020).The early efforts to develop a single stress state for unsaturated soil conditions were later followed by two independent stress state variable approaches (Matyas and Radhakrishna 1968; Fredlund and Morgenstern 1977). Although two independent stress state approach using net normal stress and matric suction has been extensively considered over last decades, several researches have proposed modified forms of effective stress for unsaturated soil conditions (e.g., Khalili and Khabbaz 1998; Nuth and Laloui 2008; Lu and Griffiths 2004; Lu and Likos 2006; Lu et al. 2010; Alsherif and McCartney 2014; Manahiloh et al. 2016).

Despite its popularity, two stress state variable approach is subjected to a major practical limitation: It cannot be utilized within the context of classical mechanics that considers effective stress as the single stress state variable governing the soil behavior (Lu et al. 2010). Moreover, smooth transition from unsaturated to saturated state in elasto-plastic models is another challenge in the use of the two independent stress state approach (Houston, 2019).

In this study, the single stress state approach by Lu et al. (2010) was utilized to calculate the effective stress field. The evolution of the effective stress field subject to climate variables was then used to anticipate the volumetric change behavior of the expansive soils using traditional effective stress-strain analysis.

Lu and Likos (2006) extended the pioneering work by Bishop (1954 and 1959), to define a new stress variable called suction stress (σ^s) within the context of Terzaghi's effective stress equation as following:

$$\sigma' = \sigma_{nt} - \sigma^s \qquad [2]$$

Lu et al. (2010) proposed a closed-form expression for suction stress for the full range of matric suction:

$$\sigma^s = -h \qquad h \leq 0 \qquad [3]$$

$$\sigma^s = \frac{h}{(1 + [\alpha\, h]^n)^{(n-1)/n}} \qquad h \geq 0 \qquad [4]$$

where α and n are the parameters used to define the soil water characteristic curve (SWCC) of a soil using van Genuchten's (1980) equation.

3 PROPOSED FORMULATION

3.1 Void ratio- effective stress relationship

The relationship of void ratio (e) versus logarithm of effective stress (σ') has been extensively utilized for saturated soils. For unsaturated soil conditions, laboratory

testing to determine soil deformation behavior is normally carried out under controlled net total stress and controlled soil suction separately. In this study, a mathematical model was developed to relate the variation of void ratio and effective stress change based on available soil constitutive data in terms of two stress variables i.e., net total stress (σ_{nt}) and soil suction (h).
Assuming that void ratio is defined as a function of net total stress and soil suction i.e., $e = f(\sigma_{nt}, h)$, the differential of this function is given by:

$$de = \frac{\partial f}{\partial \sigma_{nt}}\, d\sigma_{nt} + \frac{\partial f}{\partial h}\, dh \qquad [5]$$

The single stress state approach suggests that void ratio can be defined as a function of effective stress that is identified as $e = g(\sigma')$ in this study. Similarly, the deferential of this function can be expanded as below:

$$de = \frac{\partial g}{\partial \acute{\sigma}}\, d\acute{\sigma} = \frac{\partial g}{\partial \acute{\sigma}}\, (d\sigma_{nt} - d\sigma_s) \qquad [6]$$

The change in net normal stress, where the soil deformations under climate variables is studied, can be ignored. Moreover, net normal stress change where the soil is subject to light loads such as pavements or light buildings is normally minimal. Therefore, one can assume that the soil deformation can be largely due to changes in matric suction. In this study, the net normal stress is assumed constant in the development of the mathematical model ($d\sigma_{nt} = 0$). This assumption has been considered in the previous studies (e.g. Adem and Vanapalli, 2013). Therefore, Eq. 5 and Eq. 6 reduce to the following equations:

$$de = \frac{\partial f}{\partial h}\, dh \qquad [7]$$

$$de = -\frac{\partial g}{\partial \acute{\sigma}} d\sigma_s \qquad [8]$$

The above equations can be combined to obtain an equation for the change of a void ratio with respect to soil suction at a given total net stress:

$$\frac{de}{d\acute{\sigma}} = -\frac{\partial f}{\partial h} \times \frac{1}{\frac{d\sigma_s}{dh}} \qquad [9]$$

The first derivative of suction stress with respect to soil suction is given by (Lu et al., 2010):

$$\frac{d\sigma_s}{dh} = -\frac{1}{(1 + [\alpha h]^n)^{\frac{n-1}{n}}} + \frac{(n-1)[\alpha h]^n}{(1 + [\alpha h]^n)^{\frac{2n-1}{n}}} \qquad [10]$$

In this study, Eq. 9 was used to calculate the soil deformation modulus for effective stress-strain analysis as will be presented in the next section.

3.2 Soil modulus

A number of previous studies have utilized elastic moduli to simulate the heave behavior of expansive soils. In general, an incremental procedure using small increments of stress and strain can be used to apply the linear elasticity formulation to model the non-linear stress-strain behavior of soil (Vu, 2003). Two elastic moduli are normally employed to calculate the soil deformations induced by change in net normal stress and matric suction within the two-stress state framework. This framework was originally proposed by Fredlund and Morgenstern (1976). However, the coefficient of volume change with respect to a change in effective stress can be considered in an effective stress analysis as follows:

$$m_v = \frac{1}{1 + e_0} \frac{de}{d\acute{\sigma}} \qquad [11]$$

For K0-loading condition, the coefficient of volume change can be related to elastic modulus using:

$$E = \frac{(1 + \mu)(1 - 2\mu)}{(1 - \mu)m_v} \qquad [12]$$

where μ is soil Poisson's ratio.
The dependency of elastic modulus on effective stress is well established for the saturated soils. Under unsaturated conditions, the previous experimental studies confirm the dependency of elastic modulus on soil volumetric water content (e.g., Sawangsuriya et al. (2009); Ng et al. 2009; Schuettpelz et al. 2010 and Khosravi and McCartney 2011). It has been observed that a soil elastic modulus can increase up to several orders of magnitude from fully saturated to dry conditions due to soil water retention mechanisms i.e. adsorption and capillarity (Zhang et al., 2021)
Eq. 9, Eq. 11 and Eq.12 were used in the current study to obtain the variation of elastic modulus with soil suction at a given net total stress level. The soil water characteristic curve (SWCC) using van Genuchten's (1980) equation was utilized to calculate the elastic modulus variation with volumetric water content.
Several semi-empirical models for predicting the variation of elastic modulus with soil volumetric water content or soil matric suction under unsaturated conditions have been proposed in the literature (e.g., Sawangsuriya et al. (2009); Oh et al. (2009); Lu and Kaya (2014) and Zhang et al., 2022).
In this study, the following power law equation for soil elastic modulus as a function of volumetric water content was utilized in the numerical analysis:

$$E(\theta) = E_d + (E_w - E_d)\left(\frac{\theta - \theta_d}{\theta_w - \theta_d}\right)^m \qquad [13]$$

where $E(\theta)$ is the elastic modulus, $E(\theta) = E_d$ for the drying state when $\theta = \theta_d$ usually measured at the residual saturation state; and $E(\theta) = E_w$ for the wetting state, when $\theta = \theta_w$ usually measured at the fully saturated state; and m is an empirical parameter.

4 CASE STUDY

4.1 Problem Statement

Figure 1 illustrates a 5 m layer of Regina expansive clay which is partially covered by a flexible cover. The uncovered portion of the ground surface is subject to a constant rate equal to 1.73 mm/s. The initial matric suction is taken to be constant throughout the depth and equal to 400 kPa. This example was first simulated by Vu (2003) and has been considered in later publications (e.g. Vu and Fredlund 2006, Adem and Vanapalli 2015). The variation of soil moisture conditions and expansive clay swelling during 175 days was simulated using the developed numerical model.

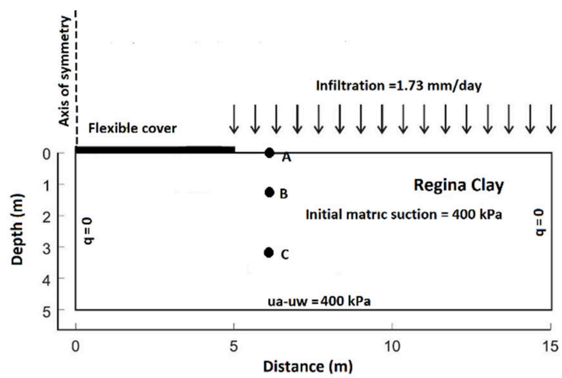

Figure 1. Geometry and hydraulic boundary conditions

4.2 Numerical Model

In this study, HYDRUS (2D/3D) version 3.04.0140 (Šimůnek et al. 2006) together with Slope Cube module (Lu et al. 2016) were utilized to simulate two-dimensional transient fields of soil moisture, soil suction, suction stress, total and effective stresses and deformation. HYDRUS 2D is a finite element software that numerically solves a modified version of the Richards' equation for movement of water flow in variably saturated soils. Slope Cube simulates stresses and deformation from transient variably saturated flow analysis from HYDRUS. Slope Cube, which was originally developed to analyze the initiation and locations of landslide occurrences under rainfall conditions, is an effective stress-based finite element module that is capable of calculating soil deformations due to changes in suction stress field as described in Section 3. The utilization of the suction stress-based effective stress to calculate the effective stress distribution and its resulting deformation in the variably-saturated soil is beyond the capabilities of typical commercial finite element software used in geotechnical engineering. Moreover, the module utilizes the power law equation to consider the variation of elastic modulus with soil moisture. The components of the hydro-mechanical numerical model have been validated in several previous studies (Lu and Godt, 2013; Lu et al. 2016).

The finite-element model consists of 10124 triangular elements. Finer mesh was generated near the unpaved surfaces to provide appropriate resolution within the infiltration zone. At each time step, the pore pressure matric suction) distribution affected by infiltration is updated by seepage analysis (HYDRUS 2D). The information is used by Cube module to calculate the suction stress and effective stress field and the resultant soil deformations.
The schematic of the model with hydraulic boundary conditions is shown in Figure 1. The finite element stress-strain analysis was carried out with the left and right boundaries free to move in the vertical direction; while the bottom boundary is fixed in both vertical and horizontal directions.

4.3 Regina Clay Parameters

Figure 2 shows the soil water characteristic curve (SWCC) and unsaturated hydraulic conductivity function (HCF) for Regina clay, obtained from Vu (2003) and Adem and Vanapalli (2013). HYDRUS Cube module utilizes α and n parameters based on SWCC curve using the mathematical model suggested by van Genuchten (1980) to calculate the suction stress. These parameters are required to calculate suction stress using Eq. 4.

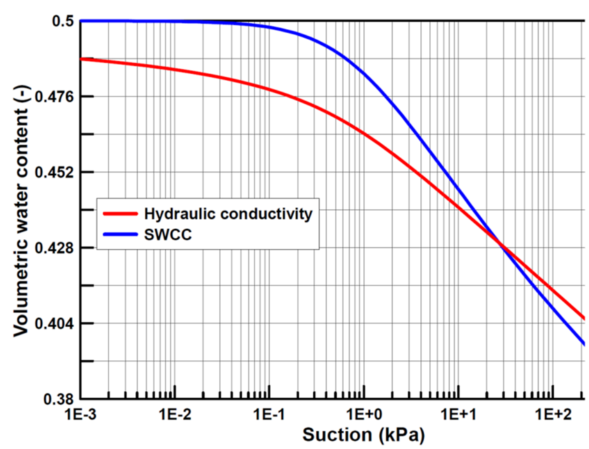

Figure 2. SWCC and HCF curves used for Regina clay in this study

Vu (2003) examined nine different equations proposed for void ratio constitutive surface (void ratio versus net normal stress and matric suction) for Regina clay using laboratory test data by Shuai (1996). He concluded that the following equation gives the best-fit surface for Regina clay:

$$e = a + b\, log\left[\frac{1 + c\sigma_{nt} + dh}{1 + f\sigma_{nt} + gh}\right] \qquad [14]$$

where a, b, c, d and f are empirical fitting parameters. Vu (2003) proposed a=1.183, b=-0.283, c=0.015, d=0.045, f=0, and g=0.00534 for Regina clay.
The soil elastic moduli for dry and saturated conditions used in this study are presented in

Table 2. These values were obtained from the best-fit power low curve matched to the calculated elastic modulus versus soil volumetric soil moisture data. It should be noted that the saturated elastic modulus for Regina clay was estimated $E_{sat}\ (MPa)$=1.1 MPa by Adem (2015) which is greater than the value used in this study.

Table 1. Regina clay- physical parameters used in this study

e_0	M	G_s	γ_t (kN/m)
0.955	0.4	2.83	17.27

Table 2. Regina clay- elastic parameters used in this study

E_d (MPa)	$E_w\ (MPa)$	m	μ
51.2	0.32	0.11	0.4

4.4 Results and Discussions

The variation of matric suction distribution as well as horizontal and vertical soil deformation profiles were computed under 175 days infiltration at the unpaved ground surface using HYDRUS Cube module.

Figure 3, Figure 5 and Figure 4 illustrate the distribution of suction head, vertical and horizontal deformations at the day 175. The infiltration caused the matric suction to reduce from its initial value of 400 kPa (40.78 m) up to approximately 6 m at the vicinity of the unpaved ground surface. As one can expect, the decrease in the matric suction resulted in soil heaving. The maximum heave after 175 days is approximately 50 mm which occurred at the farthest left side of the domain at the ground surface. In addition to heaving, the asymmetric infiltration conditions caused lateral movements in the expansive soil. The numerical results indicate that the maximum horizontal deformation at the corner of the paved area is approximately 13 mm. The obtained values are in good agreement with those reported by Vu (2003).

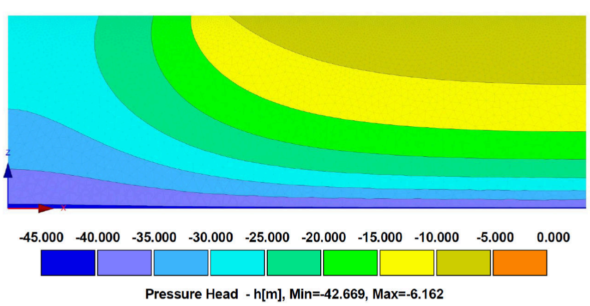

Figure 3. Suction head distribution at day 175

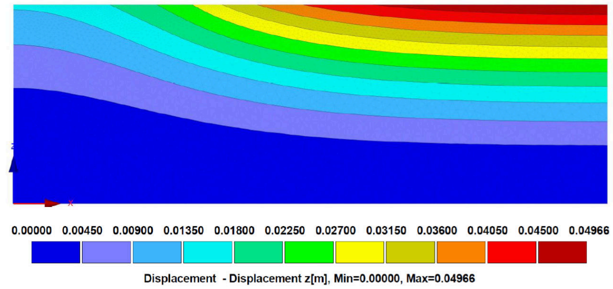

Figure 4. Vertical displacement distribution at day 175

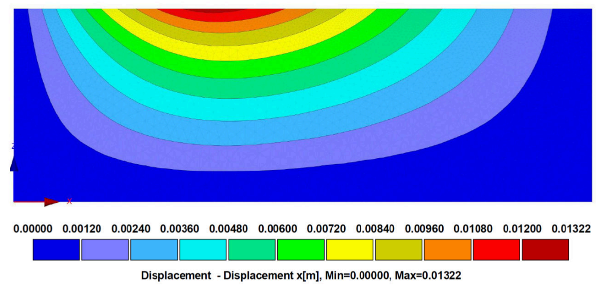

Figure 5. Horizontal displacement distribution at day 175

Figure 6 and Figure 7 illustrate the comparison of predicted heave using the developed numerical model and the results obtained by Vu and Fredlund (2006) using a coupled approach at the three locations (Nodes A, B, and C) over time. The observation nodes A, B and C are located 1m away from the paved area at depth 0, 1.5 m and 3.5 m, respectively. Although the anticipated matric suction at node C is approximately 20% less than what was reported by Vu and Fredlund (2006), the matric suction compares well at shallower depths (Nodes A and B). The anticipated variation of heave with time indicates good agreement between two studies. The heave profile with depth (Figure 8) and the ground surface profile (Figure 9) at three different days i.e., day 13, day 53 and day 175 also confirm the good agreement between the results obtained from two studies despite different unsaturated soil mechanics formulations. The results show that more than 75% of the total heave is completed over 53 days.

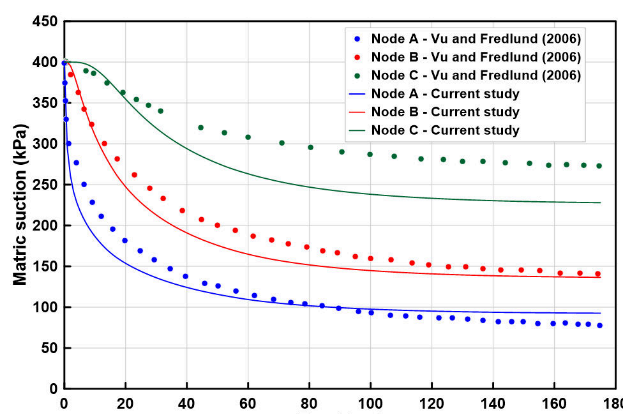

Figure 6. Matric suction with time at points A, B, C

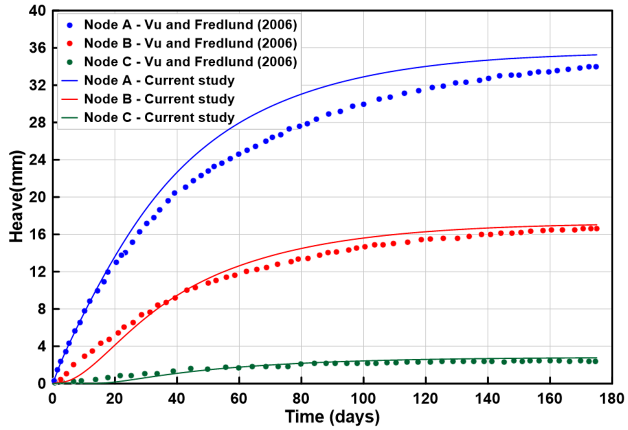

Figure 7. Vertical displacement (heave) with time at points A, B, C

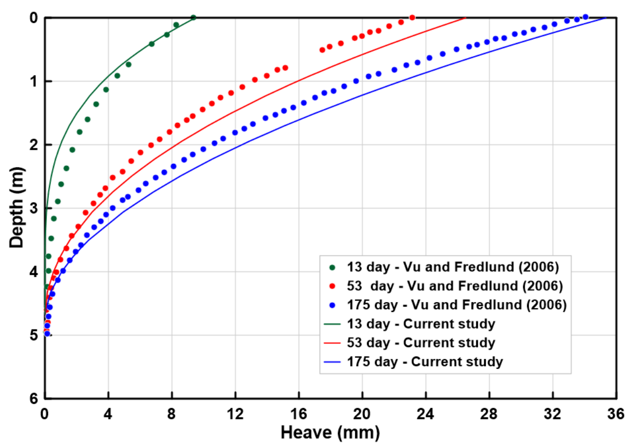

Figure 8. Vertical displacement (heave) with depth at different times

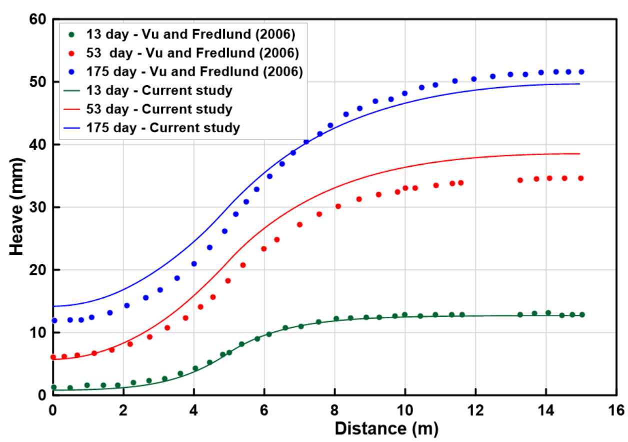

Figure 9. Ground surface heave profile at different times

Adem and Vanapalli (2013) carried out a comparison study for estimating the heave for the same benchmark problem using three different methods: Hamberg (1985), Zhang and Briaud (2010) and MEBM. Table 3 presents a summary of the maximum heave at ground surface (Node A) after 175 days using these three methods and the results obtained from effective stressed based analysis in this study. It should be noted that these three methods consider one dimensional heave, whereas the numerical model of Vu and Fredlund (2006) and the numerical model developed in the current study include two-dimensional deformation analysis. And results from the current study compare better to Vu and Fredlund (2006) by using fewer fitting parameters.

Table 3. Maximum heave at Node A after day 175

Method / Study	Maximum Heave (mm)	Comparison*
Vu and Fredlund (2006)	34	1.00
Adem an Vanapalli (2013)	41	1.21
Hamberg (1985)	71	2.09
Zhang and Briaud (2010)	58	1.71
This study	35	1.03

* with respect to Vu and Fredund (2006)

5 CONCLUSION

In this study, two-dimensional finite element analysis was employed to simulate transient variably-saturated seepage analysis and estimate soil deformations under mechanical and environmental loadings using elastic-based effective stress-strain analysis. The effective stress field is updated based on the changes in suction stress which is a function of matric suction and soil water characteristic curve. A mathematical approach was utilized to obtain the elastic modulus from the void ratio constitutive surface data. A power law equation was then employed to incorporate the dependency of soil modulus on volumetric water content of soil in the analysis.

The developed numerical model was used to model the problem of a Regina expansive clay layer, which is partially covered by a flexible pavement, subjected to a constant infiltration rate over 175 days. The results obtained indicate the capability of the proposed model in simulation of expansive soil deformations subject to varying moisture conditions over time. In contrast to the available one-dimensional methods, the proposed model can simulate the two-dimensional distribution of suction head, vertical and horizontal soil deformations with time. Such vertical and horizontal soil deformations can cause significant differential movements over time. The numerical results are in good agreement with those reported by Vu and Fredlund (2006) with fewer fitting parameters.

The obtained results indicate the capability of the proposed model in simulation of expansive soil deformations under climate data over time. The validation of the developed model against various soil-atmosphere conditions such as participation and evaporation over long term climate data considering climate change is part of the ongoing research.

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