Constitutive modelling of unsaturated tailings undergoing drying and wetting cycles

Mina Mofrad, Mehdi Pouragha, & Paul H. Simms
Department of Civil & Environmental Engineering – Carleton University, Ottawa, ON, Canada

GeoCalgary 2022 October 2-5
Reflection on Resources

ABSTRACT

The current study reviews and compares some of the commonly used constitutive models for partially saturated soils, namely Barcelona Basic Model (BBM), Glasgow Coupled Model (GCM), as well as the model developed by Buscarnera and Nova (BNM). The latter has been improved herein by incorporating the possibility of irreversible hydraulic mechanisms in saturation cycles. The effect of modelling elements such as suction induced yielding, shear hardening, non-associated plasticity, and hydraulic hysteresis are explored. In particular, we investigate the advantages and disadvantages of each model and demonstrate the gradual increase in their capability of capturing the salient trends as the models become more complex. The models have been used to reproduce the experimental results on an unsaturated gold mine tailings that experienced consecutive cycles of wetting and drying.

RÉSUMÉ

La présente étude examine et compare certains des modèles constitutifs couramment utilisés pour les sols partiellement saturés, à savoir le modèle de base de Barcelone (BBM), le modèle couplé de Glasgow (GCM), ainsi que le modèle développé par Buscarnera et Nova (BNM). Ce dernier a été amélioré ici en incorporant la possibilité de mécanismes hydrauliques irréversibles dans les cycles de saturation. L'effet des éléments de modélisation tels que le rendement induit par l'aspiration, l'écrouissage par cisaillement, la plasticité non associée et l'hystérésis hydraulique est exploré. En particulier, nous étudions les avantages et les inconvénients de chaque modèle et démontrons l'augmentation progressive de leur capacité à capturer les tendances saillantes à mesure que les modèles deviennent plus complexes. Les modèles ont été utilisés pour reproduire les résultats expérimentaux sur des résidus de mine d'or non saturés qui ont subi des cycles consécutifs de mouillage et de séchage.

1 INTRODUCTION

The waste byproducts of mining processes, also known as tailings, are often deposited in impoundments whose structural integrity poses critical geotechnical and geo-environmental challenges (Simms 2017). Additional to the usual mechanical loads associated with such layered embankments, tailings deposits also undergo complex hydro-mechanical loading history due to consecutive desiccation and saturation cycles. Such stress histories in multilayer deposition involve self-weight settling, followed by desiccation, which is in turn followed by another cycle of rewetting due to the placement of new layers.

For the most part of their loading history, tailings exist in a partly saturated, triphasic state consisting of air, water, and solid. The behaviour of these unsaturated material naturally transcends the conventional soil mechanics that is established for soils in dry or saturated conditions. Contemporary unsaturated soil mechanic extends the traditional theories to include new physics such as matric suction, coupling of hydraulic and mechanical behaviours, and hysteretic effects during cycles of drying and wetting (Fredlund 1993,Lu and Likos 2004).

In particular, the concept of effective stress has been extended to include additional variables such as matric suction (Bishop 1959). However, unlike the saturated case, the behaviour of unsaturated soils has been shown not to be reducible to a single effective stress parameter (Duriez et al. 2018). As such, other components of constitutive models are also further elaborated to properly address the unsaturated soil behaviours. This led to a new branch of constitutive modelling in soil mechanics for partially saturated soil.

Amongst the first such constitutive models is the Barcelona Basic Model (BBM) (Alonso et al. 1990) which extends the Modified Cam Clay Model (MCC) (Roscoe and Burland 1968) to unsaturated soils by incorporating the matric suction as a state variable on which the yield surface depends. The model was later on improved by Lloret-Cabot et al. (2013) who, starting from the thermodynamic framework by Houlsby (1997), formulated the so-called Glasgow Coupled Model (GCM).

The current study explores the performance of the commonly used constitutive models in predicting the behaviour of partially saturated tailing materials. A systematic comparison is carried out between the BBM and GCM. Moreover, the possibility of having shear induced hardening and non-associated plastic flow is also investigated by considering by Buscarnera and Nova (2009) model (BNM). The latter has been improved here by incorporating the possibility of hydraulic hysteresis on closed wetting/drying cycles. The models are calibrated and used to reproduce the influence of stress history, including drying and rewetting, on behaviour of thickened gold tailings (Daliri et al. 2014, 2016).

2 CONSTITUTIVE MODELS FOR UNSATURATED SOILS

In this section, we briefly review the most relevant parts of the three constitutive models discussed in this paper: BBM, GCM, and BNM. In the following formulations, the hydrostatic and the deviatoric components of stress tensors, and their conjugate strains are defined as follows:

$$p = \frac{\sigma_{ii}}{3}, \qquad q = \sqrt{\frac{3}{2} s_{ij} s_{ij}}, \qquad s_{ij} = \sigma_{ij} - p\delta_{ij} \qquad [1]$$

$$\varepsilon_v = \varepsilon_{ii}, \qquad \varepsilon_s = \sqrt{\frac{2}{3} e_{ij} e_{ij}}, \qquad e_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon_v \delta_{ij} \qquad [2]$$

With $\boldsymbol{\delta}$ being the Kronecker delta and repeated indices denoting summation. The over dot ($\dot{X}$) represents incremental rate.

2.1 Barcelona Basic Model (BBM)

The Barcelona Basic Model extends the MCC by assuming the critical state (and hence the yielding) of the soil to depend directly on suction (Alonso et al. 1990). Despite of being based on MCC model, which is primarily developed for clays, the BBM is claimed to be applicable to slightly or moderately expansive soils including partially saturated sands, silts, clayey sands, and sandy clays. The model considers two stress state variables, the net pressure ($\sigma'_{ij} = \sigma_{ij} - u_a \delta_{ij}$) and matric suction ($s = u_a - u_w$), with $\boldsymbol{\sigma}$ being the total stress tensor, and u_a and u_w being the air and water pressures. The yield function takes an elliptical form similar to the MCC:

$$f_1 := q^2 - M^2(p' + p_s)(p_p - p') = 0 \qquad [3]$$

where M is the $q - p$ slope at critical state. The variables p_s and p_p depend on suction as:

$$p_s = \ a\, s, \quad p_p = p^c \left(\frac{p_{p_0}}{p^c}\right)^{\frac{(\lambda_0 - \kappa)}{(\lambda(s) - \kappa)}} \qquad [4]$$

where a, p^c, λ_0, and κ are material constants, p_{p_0} is the hardening parameter. in Eq. 4, $\lambda(s)$ is the stiffness parameter that also depends on matric suction:

$$\lambda(s) = \lambda_0[(1 - r)\, exp(-\beta s) + r] \qquad [5]$$

with r and β being model constants. Figure 1 presents the general form of the yield surface in the $p' - q - s$ space. The model can be interpreted as having two coupled yield surfaces. According to Equation [4], the size of the elastic domain increases with higher suction values and spans from $-p_s$ to p_p along p' axis.

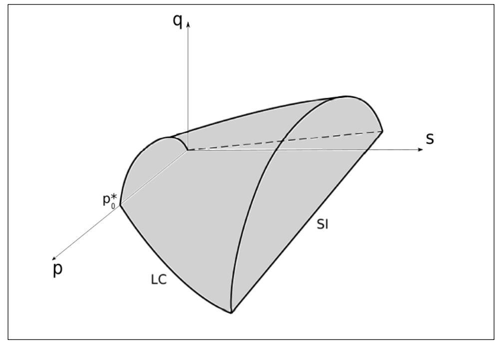

Figure 1. Yield surfaces of BBM in (p', q, s) stress space.

The second yield surface is the vertical cap that depends on the maximum historic suction value previously experienced, s_0.

$$f_2 := s - s_0 = 0 \qquad [6]$$

The hardening in both yield surfaces occurs with respect to the plastic volumetric strain, $\dot{\varepsilon}_v^p$:

$$\dot{p}_{p_0} = p_{p_0} \frac{(1+e)}{\lambda_0 - \kappa} \dot{\varepsilon}_v^p \qquad [7]$$

$$\dot{s}_0 = (s_0 + p_{atm}) \frac{(1+e)}{\lambda_s - \kappa_s}\, \dot{\varepsilon}_v^p \qquad [8]$$

where λ_s and κ_s are model parameters, e is the void ratio, and p_{atm} is the atmospheric pressure. The BBM assumes associated plasticity for all yield surfaces. It is seen that in BBM formulation, the suction is treated as a state variable with no clear energy conjugate. This is rectified in GCM model.

2.2 Glasgow Coupled Model (GCM)

GCM (Wheeler et al. 2003, Lloret-Cabot et al. 2013) differs from BBM mainly in that it considers the suction and saturation as energy conjugates that allows capturing hydraulic hysteresis. The stress state is characterized by Bishop's stress ($\sigma^*_{ij} = \sigma_{ij} - (S_r\, u_w + (1 - S_r)\, u_a)\delta_{ij}$) and modified suction ($s^* = n\,(u_a - u_w)$) where S_r is the degree of saturation and n is the porosity. Similar to the BBM, associated plasticity is used in GCM formulation.

The yield surface assumes a cylindrical form in the $p^* - q - s^*$ space described by three yield surfaces:

$$f_1 := q^2 - M^2 p^*(p_0^* - p^*) = 0 \qquad [9]$$

$$f_2 := s^* - s_I^* = 0, \quad f_3 := s_D^* - s^* = 0 \qquad [10]$$

Figure 2 shows yield surface of the GCM in $p^* - q - s^*$ space. Plastic strains occur during yielding on mechanical curve, whereas plastic variations of S_r occur during yielding along SD and SI surfaces.

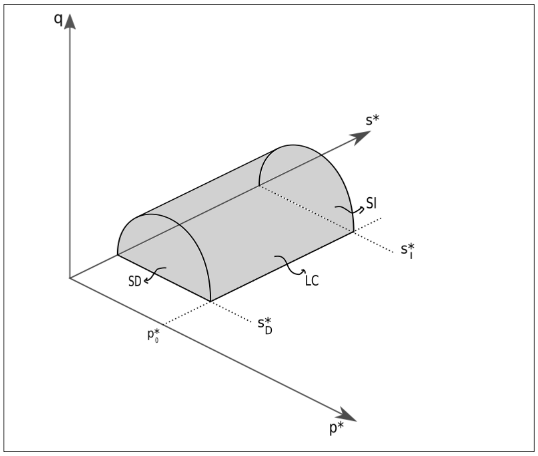

Figure 2. Yield surfaces of GCM.

The evolution of hardening parameters $\dot{p}_0^*$, $\dot{s}_I^*$ and $\dot{s}_D^*$, are expressed interms of plastic increments of volumetric strain $\dot{\varepsilon}_v^p$, and plastic increments of degree of saturation $\dot{S}_r^p$.

$$\dot{p}_0^* = \dot{p}_0^* \left(\frac{(1+e)}{\lambda_0 - \kappa} \dot{\varepsilon}_v^p - \frac{k_1}{\lambda_s - \kappa_s} \dot{S}_r^p \right) \qquad [11]$$

$$\dot{s}_x^* = s_x^* \left[-\frac{\dot{Sr}^p}{\lambda_s - \kappa_s} + \frac{k_2(1+e)}{\lambda_0 - \kappa} \dot{\varepsilon}_v^p \right], \quad x = I \ or \ D \qquad [12]$$

Explicitly considering s^* and $-S_r$ as energy conjugates allows GCM to capture more consistently the transition between saturated and unsaturated states, as well as to better include the influence of retention hysteresis. The form of Equation 12 implies that the suction-saturation relation will assume a log-linear form (linear in semi-log space). The consequences of this assumption will be discussed in the next sections.

2.3 Extended Buscarnera and Nova Model (BNM)

Similar to the GCM, the model developed by Buscarnera and Nova (2009) adopts the Bishop's stress and modified suction as the main variables as suggested by Houlsby (1997). However, instead of starting from MCC formulation, BNM starts from the dilatancy relation and derives a plastic potential incorporating suction. Unlike BBM and GCM, the plastic flow is non-associated with the yield surface taking the same functional form as the plastic potential with different coefficients. With a formulation based on dilatancy and non-associated plasticity, the BNM is a better candidate for capturing the behaviour of unsaturated frictional soils and it has been used to explore the instability mechanisms in such geomaterials (Buscarnera and Nova 2009).

The yield surface (f) and the plastic potential (g) of the BNM are expressed as:

$$f = \left[A_f^{\frac{k_1 f}{C_f}} B_f^{-\frac{k_2 f}{C_f}} \right] p^* - p_s^* = 0 \qquad [13]$$

$$g = \left[A_g^{\frac{k_1 g}{C_g}} B_g^{-\frac{k_2 g}{C_g}} \right] p^* - p_s^* = 0 \qquad [14]$$

with k_1 and k_2 being constant material parameters, and A, B, and C being coefficients that depend on the current stress state. The shape of the yield function and the plastic potential is controlled mainly by the single hardening parameter p_s^*, which, unlike the other two models, depends on both volumetric and shear plastic strains as follows.

$$\dot{p}_s^* = \frac{p_s^*}{B_p} \left(\dot{\varepsilon}_v^p + \xi_s \dot{\varepsilon}_s^p \right) - r_{sw} p_s^* \ \dot{S}_r \qquad [15]$$

where B_p, ξ_s, and r_{sw} are model constants.

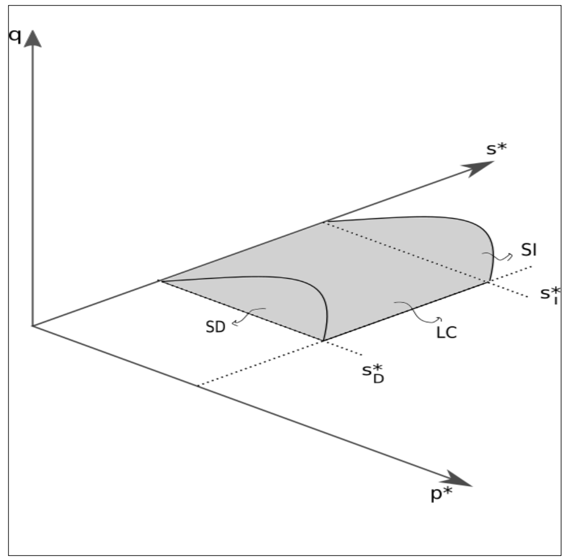

Figure 3. Yield surfaces of the extended BNM.

The BNM considers only one mechanical yield surface, and as such, the coupled plastic effect of hydraulic and mechanical processes is neglected. The current study improves BNM by adopting the same approach as GCM; the yield surface is extended in third dimension of s^* with two additional caps describing the hydraulic plastic yielding:

$$s^* - s_I^* = 0, \quad s_D^* - s^* = 0 \qquad [16]$$

which enables the model to capture irreversible change of saturation ratio ($\dot{Sr}^p$). Figure 3 shows the extended 3D yield surface in $p^* - q - s^*$ space. The hardening associated with the two hydraulic caps is assumed to be the same as in the GCM model given in Equation [12]. In the original model the suction-saturation relation is given by the van Genuchten model (Van Genuchten 1980). However, assuming Equation [12] indicates that, similar to the GCM model, the suction-saturation relation assumes a log-linear form with different slopes along hydraulic loading and unloading. Table 1 compares the main elements of these models.

Table 1: Characteristics of the three considered models.

Characteristics/Model	BBM	GCM	BNM-original	BNM
Stress variable	σ', s	σ^*, s^*	σ^*, s^*	σ^*, s^*
Hydraulic energy conjugates		✓	✓	✓
Coupled yield surfaces	✓	✓		✓
Non-associated plasticity	✓		✓	✓
Hydro-mechanical hardening		✓		✓
Hydraulic hysteresis		✓		✓

3 RESULTS

The three described models have been implemented numerically using explicit formulation and are applied to predict an experimental dataset for unsaturated soils. We focus on the experimental study by Daliri et al. (2014) and who studied the effect of stress and saturation history on behaviour of unsaturated gold tailings. Soil specimens were first prepared in a slurry form and deposited in two layers. The bottom layer experienced the initial self-weight settling followed by desiccation to three different gravimetric water contents, $W_d = 23\%, 17\%, 12\%$, and then rewetted to saturated condition once the second layer was placed. Soil samples taken from the bottom layer were tested in a simple shear apparatus in undrained condition subjected to vertical stress of $50\ kPa$.

The testing results during the desiccation/rewetting and the shearing process were used to calibrate the three constitutive models with the parameters given below.

- BBM (10 parameters): $\nu = 0.3, \kappa = 0.042, \kappa_s = 0.0015, \lambda = 0.126, \lambda_s = 0.052, M = 0.6, \alpha = 0.1, \beta = 0.07\ kPa^{-1}, r = 0.7, p^c = 10\ kPa$.
- GCM (8 parameters): $\nu = 0.3, \kappa = 0.01, \kappa_s = 0.08, \lambda = 0.12, \lambda_s = 0.14, M = 0.6, k_1 = 0.3, k_2 = 0.4$.
- BNM (17 parameters): $\nu = 0.3, \kappa = 0.01, \kappa_s = 0.08, \lambda = 0.12, \lambda_s = 0.14, k_2 = 0.45, B_p = 0.04, \xi_s = 0.65, r_{sw} = 1.5, \alpha_f = 0.45, m_f = 0.9, M_{cf} = 0.6, M_{ef} = 0.6, \alpha_g = 0.4, m_g = 0.9, M_{cg} = 1, M_{eg} = 1$.

The critical state parameters were found by considering the stress state and void ratio at the end of shear tests and the parameters capturing the effect saturation were calibrated by trial and error on both drying/wetting cycles and shear test responses. Note that the GCM model has been previously calibrated for the same material undergoing self-weight consolidation and wetting-drying cycles in the drying box (Qi et al. 2020). The calibrated parameters are generally close to the values obtained here. However, the shearing tests results included herein lead to slightly different values.

Figure 4 and 5 compare the predictions by the three models with the experimental results shown as square symbols. The figures include the change in void ratio vs. suction and net normal stress for two samples (Figure 4), and shear stress vs. shear strain together with the effective stress path (Figure 5).

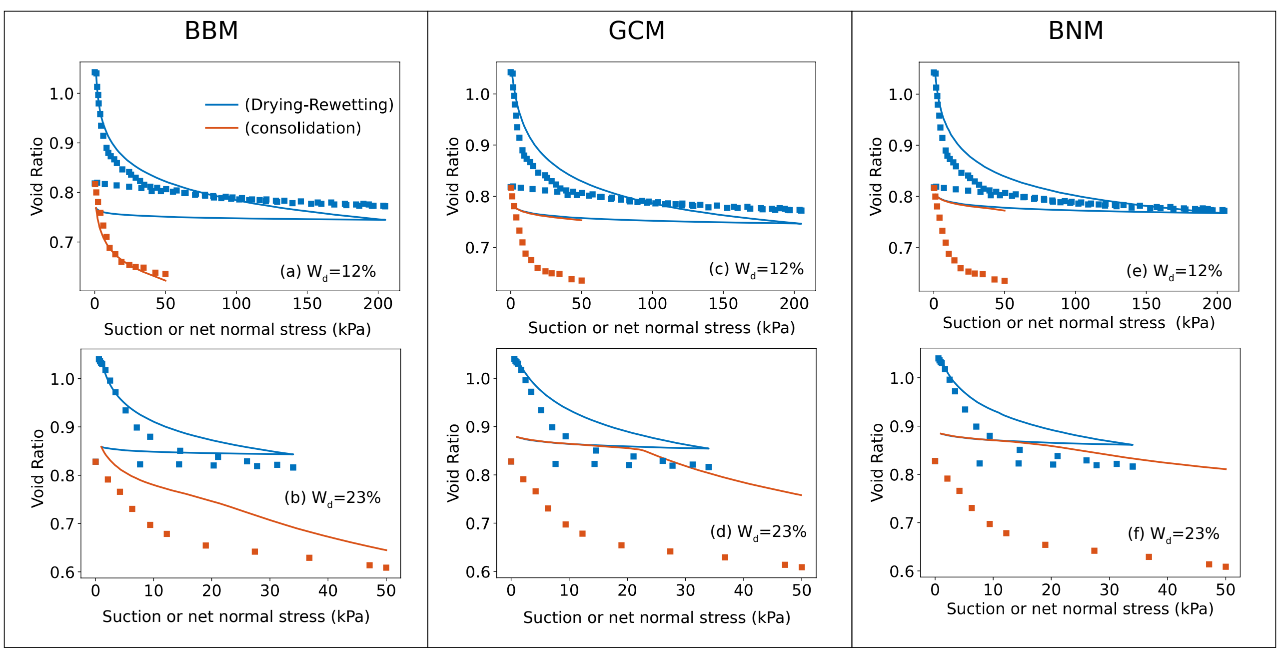

Figure 4: Comparison between the experimental results and the model predictions for volume change. Samples are desiccated to $W_d = 12\%$ (a, c, e), or $W_d = 23\%$ (b, d, f), then rewetted to fully saturated state, and consolidated to $50\ kPa$. The square symbols represent the experimental data and solid lines are model results.

4 DISCUSSION

The performance of the three constitutive models, BBM, GCM, and BNM are compared in predicting the shear response of desiccated/rewetted gold tailings. Focusing on the void ratio changes during the desiccation, rewetting, and consolidation, we observe that only the BBM model provides realistic trends. The GCM and BNM models underestimate the compaction upon rewetting and consolidation. This is attributed to the relatively high preconsolidation stress induced by these models during the desiccation stage. The inclusion of hydraulic yield surfaces causes the sample to enter the elastic zone during the initial suction increase, and as such, the consolidation stage will be primarily located within the elastic domain.

Recalling the model construction in Figure 2, we see that the GCM model (and the extended BNM model) does include the capability of yielding upon wetting (suction decrease) when $s_D \neq 0$. However, according to Equation [12], the change in s_D is proportional to its current value (log-linear) and as such, the initially small value of s_D results in negligible increase in its value during the test. This error can be attributed to the replacing the suction-saturation relation with a single log-linear curve which introduces errors at the two tails of suction. Therefore, we expect the predictions to improve if a more realistic suction-saturation relation is adopted. The BBM does not suffer from this shortcoming because the lack of coupling between the two yield surfaces results in a smaller preconsolidation stress upon desiccation.

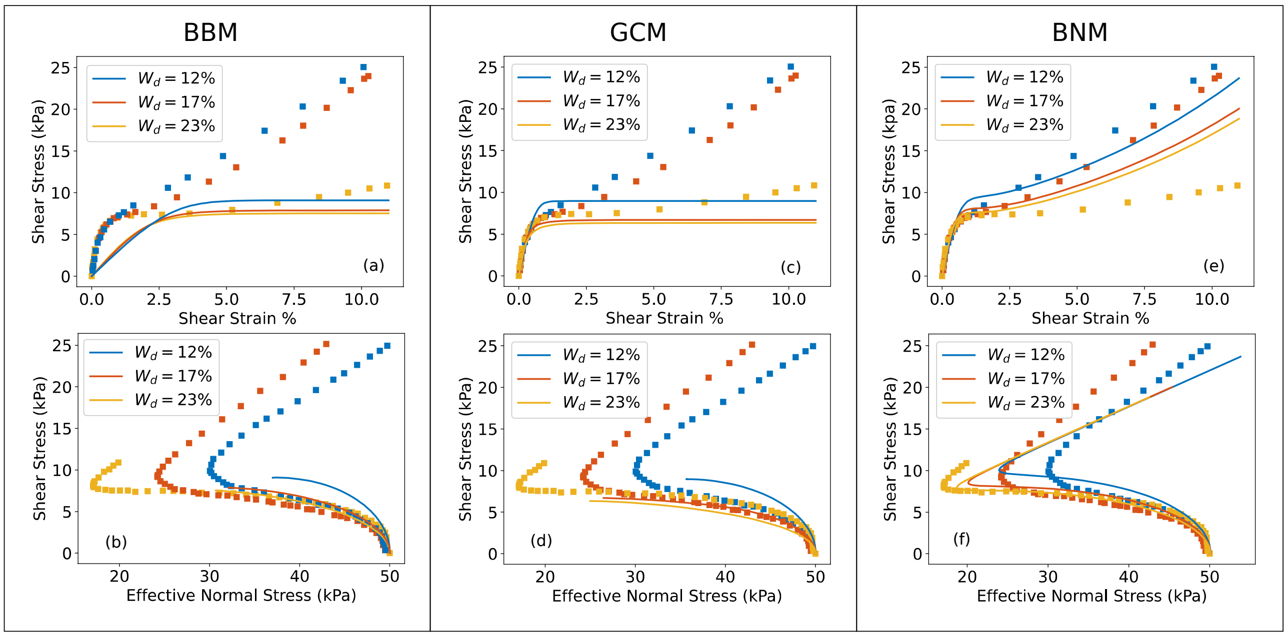

Figure 5. Comparison between the experimental results and the model predictions; (a, c, e) shear stress vs strain, and (b, d, f) simple shear effective stress path. The square symbols represent the experimental data and solid lines are model results.

Turning to the shearing results, the BBM is able to capture the general trend of increase in shear strength as desiccation intensifies. However, it fails to correctly predict the effective normal stress at which the failure stress is reached (end of solid lines). This is improved in the GCM model where the failure line is reached at lower effective normal stresses that are more reflective of the phase transformation observed in the experimental results.

Also, the BBM fails to capture the initial slope of the stress-strain curve (Figure 6a) because in the absence of hydraulic yielding, the sample will initially be at the yielding limit exhibiting plastic deformations. Recall that by keeping the sample state at the yielding limit, the BBM was able to produce a more realistic compaction during consolidation (Figure 4a & b). However, the same property now causes the initial slope not to match the experimental results.

Compared to BBM, GCM shows slight quantitative improvements in capturing the initial slope of the stress-strain curve, as well as in predicting the gradual increase in shear strength increase. This is due to the possibility of hydraulic yielding which returns the initial state of the sample within the elastic zone. However, again, the same property caused the volume change during the consolidation to be unrealistic (Figure 4c & d).

Nevertheless, BBM and GCM fail to capture the apparent hardening of the stress path where, after going through the phase transformation state, the value of normal and shear stresses constantly increases. This is primarily due to the structure of the Cam Clay based models for which the critical state void ratio and stress ratio are reached simultaneously. Therefore, with a sustained shear strain, no changes in stresses (hardening) will occur beyond this point.

The BNM resolve this issue by including a shear hardening term into Equation [15] and therefore, any shear strain, regardless of the state of the material with respect to the critical state, will induce hardening. As a result, a non-zero value of ξ_s can capture the hardening observed in the experiments. However, this comes at the cost of not having a proper critical state with zero volume change. In fact, with non-zero values of ξ_s the model predicts continued volume change even at large shear strains in drained tests, which is not consistent with the concept of critical state. The issue can be resolved by resorting to more meaningful hardening laws where separate evolutions for void ratio and stress ratio are allowed while also incorporating the critical state in terms of both stresses and void ratio (Wan and Guo 1998).

The BNM predictions are also more realistic due to its non-associated plastic flow, compare to associated flow rules in BBM and GCM. This provides more degrees of freedom in calibrating the model to the experimental results.

Finally, the comparison between the models should be understood only in the context of their complexity; the improvements in BNM modelling come at the cost of including 17 model parameters which is more than double the number of parameters for GCM. It appears that the same hydraulic conjugates can be added to other simpler and more robust models to accurately capture the behaviour of unsaturated materials while keeping model parameters manageable. Instances of such works can be found in the recent works such as Fern et al. (2016) and Liu and Muraleetharan (2012). However, the complexity of the formulation in such models have so far prevented them from being used in engineering practice.

5 CONCLUSION

We reviewed in this study three commonly used constitutive models for unsaturated materials and compared their performance in predicting the hydro-mechanical behaviour of gold tailings subjected to desiccation/rewetting cycles and simple shear loading. Comparison with the experimental results indicate that while BBM model produces a more realistic compressive volume change during consolidation, it underestimates the initial stiffness as the sample reaches the yield surfaces during the consolidation stage. The GCM on the other hand, captures the initial stiffness through its hydraulic yielding mechanism, which is nonetheless accompanied by a poor prediction of the volume change during the consolidation.

Whereas both BBM and GCM models fail to capture the phase transformation and the hardening phase, the modified version of BNM is shown to predict the continuous increase in stress beyond the phase transformation. However, this is handled by introducing unrealistic unceasing shear hardening terms that fail to capture void ratio at the critical state.

Our results imply that it should be possible to adopt the advantageous characteristics of GCM framework and apply it to a more robust constitutive model to better capture the hydro-mechanical behaviour of unsaturated soils.

Nevertheless, the comparisons between the BBM and GCM models indicates that the hydro-mechanical hardening of soils requires coupled features beyond the simple ones included in GCM. In this regard, we expect that adopting a more accurate suction-saturation relation will increase the accuracy of the GCM modelling framework.

6 REFERENCES

Alonso, E.E., Gens, A., and Josa, A. 1990. A constitutive model for partially saturated soils. Géotechnique, **40**(3): 405–430. Thomas Telford Ltd.

Bishop, A.W. 1959. The principle of effective stress. Teknisk ukeblad, **39**: 859–863.

Buscarnera, G., and Nova, R. 2009. An elastoplastic strainhardening model for soil allowing for hydraulic bonding–debonding effects. International journal for numerical and analytical methods in geomechanics, **33**(8): 1055–1086. Wiley Online Library.

Daliri, F., Kim, H., Simms, P., and Sivathayalan, S. 2014. Impact of desiccation on monotonic and cyclic shear strength of thickened gold tailings. Journal of Geotechnical and Geoenvironmental Engineering, **140**(9): 04014048. American Society of Civil Engineers.

Daliri, F., Simms, P., and Sivathayalan, S. 2016. Shear and dewatering behaviour of densified gold tailings in a laboratory simulation of multi-layer deposition. Canadian Geotechnical Journal, **53**(8): 1246–1257. NRC Research Press.

Duriez, J., Wan, R., Pouragha, M., and Darve, F. 2018. Revisiting the existence of an effective stress for wet granular soils with micromechanics. International Journal for Numerical and Analytical Methods in Geomechanics, **42**(8): 959–978. Wiley Online Library.

Fern, E.J., Robert, D.J., and Soga, K. 2016. Modeling the stress-dilatancy relationship of unsaturated silica sand in triaxial compression tests. Journal of Geotechnical and Geoenvironmental Engineering, **142**(11): 04016055. American Society of Civil Engineers.

Fredlund, D.G. 1993. Soil mechanics for unsaturated soils. John Wiley & Sons.

Houlsby, G. 1997. The work input to an unsaturated granular material. Géotechnique, **47**(1): 193–196. Thomas Telford Ltd.

Liu, C., and Muraleetharan, K.K. 2012. Coupled hydro-mechanical elastoplastic constitutive model for unsaturated sands and silts. I: Formulation. International Journal of Geomechanics, **12**(3): 239–247. American Society of Civil Engineers.

Lloret-Cabot, M., Sánchez, M., and Wheeler, S.J. 2013. Formulation of a three-dimensional constitutive model for unsaturated soils incorporating mechanical–water retention couplings. International Journal for Numerical and Analytical Methods in Geomechanics, **37**(17): 3008–3035. Wiley Online Library.

Lu, N., and Likos, W.J. 2004. Unsaturated Soil Mechanics. J. Wiley.

Qi, S., Simms, P., Daliri, F., and Vanapalli, S. 2020. Coupling elasto-plastic behaviour of unsaturated soils with piecewise linear large-strain consolidation. Géotechnique, **70**(6): 518–537. Thomas Telford Ltd.

Roscoe, K., and Burland, J. 1968. On the generalized stress-strain behaviour of wet clay.

Simms, P. 2017. 2013 Colloquium of the Canadian Geotechnical Society: Geotechnical and geoenvironmental behaviour of high-density tailings. Canadian Geotechnical Journal, **54**(4): 455–468. NRC Research Press.

Van Genuchten, M.T. 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil science society of America journal, **44**(5): 892–898. Wiley Online Library.

Wan, R., and Guo, P. 1998. A simple constitutive model for granular soils: modified stress-dilatancy approach. Computers and Geotechnics, **22**(2): 109–133. Elsevier.

Wheeler, S., Sharma, R., and Buisson, M. 2003. Coupling of hydraulic hysteresis and stress–strain behaviour in unsaturated soils. Géotechnique, **53**(1): 41–54. Thomas Telford Ltd.