Integrating geological and geotechnical variability to develop a probabilistic 3D shear-wave velocity model for postglacial soils in Saguenay, Québec

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ABSTRACT
A probabilistic approach is proposed for integrating geological and geotechnical information into the development of a 3D shear-wave velocity model and assessing the associated uncertainties. The method is applied to the Saguenay region, where subsurface geology is heterogeneous and soil sediments are varied in thickness and stiffness. A 3D geological model of the unconsolidated deposits is first developed using geostatistical interpolations and sequential indicator simulations. Seismic cone penetration tests are then conducted to develop site-specific empirical CPT-V_s and V_s-depth correlations for postglacial sediments. Nonlinear regression analyzes are conducted based on the soil types incorporating the cone tip resistance and depth for clay-like and sand-like soils. The final 3D distribution of V_s is estimated by combining the V_s-depth correlations with the likelihood of soil type occurrences. Additionally, propagated uncertainty is quantified by integrating the simulation variance of the probabilistic geological model and the statistical variance of the V_s-depth correlations.
Keywords: 3D model, seismic cone penetration test (SCPT), shear-wave velocity, uncertainty

RÉSUMÉ
Une approche probabiliste est proposée pour intégrer des informations géologiques et géotechniques dans le développement d'un modèle 3D de vitesse d'onde de cisaillement et l'évaluation des incertitudes associées. La méthode est appliquée au territoire du Saguenay, où la géologie du sous-sol est hétérogène et les sédiments sont variés en épaisseur et en rigidité. Un modèle géologique 3D des dépôts non consolidés est d'abord développé à l'aide d'interpolations géostatistiques et de simulations d'indicateurs séquentiels. Des tests de pénétration de cône sismique sont ensuite effectués pour développer des corrélations empiriques CPT-V_s et V_s-profondeur spécifiques au site pour les sédiments postglaciaires. Les analyses de régression non linéaire sont effectuées sur la base des types de sol incorporant la résistance et la profondeur de la pointe du cône pour les sols argileux et sableux. La distribution 3D finale de V_s est estimée en combinant les corrélations V_s -profondeur avec la probabilité d'occurrences de type de sol. De plus, l'incertitude propagée est quantifiée en intégrant la variance de simulation du modèle géologique probabiliste et la variance statistique des corrélations V_s -profondeur.

1 INTRODUCTION

Local site conditions tend to modify the amplitude and frequency of incoming seismic waves (Seed et al., 1976). This phenomenon is known as the seismic site effect, and it depends on the geotechnical (e.g., soil type, shear modulus, damping ratio) and geological (e.g., stratigraphy, basin topography, thickness) properties of soil sediments. The time-averaged shear-wave velocity of the top 30 m ($V_{s,30}$) is one of the well accepted proxies for seismic microzonation mapping (SM Working Group 2015; Licata et al. 2019; Molnar et al. 2020). Although shear-wave velocity (V_s) is recognized as a simple, effective and representative parameter for determining site effects, obtaining sufficient direct V_s measurements in regional site characterization studies is challenging. As a proxy, the available geotechnical data represent a useful data source for estimating V_s (Oliveira et al. 2020). In this case, empirical V_s correlations with geotechnical parameters (Salsabili et al. 2022) or depth (Motazedian et al. 2011, Podestá et al. 2019) are suggested for addressing the scarcity of spatial distribution of V_s measurements.

Geospatial modeling can be achieved using spatial variability. Spatial variation refers to the dissimilarity of pair values of a random variable as a function of distance (Isaaks and Srivastava 1989). The spatial variation in soil properties has been modeled using random field theory, which decomposes the spatial variation into a deterministic trend function and its residuals (Fenton 1999, Fenton and Griffiths 2003). This method can also be used to address problems with sparse and nonstationary data (Wang et al., 2018; Zhao and Wang, 2020). In recent soil engineering practices, geostatistical methods have also been used to predict spatially-correlated geotechnical properties, such as cone resistance and V_s (Vessia et al. 2020; Hallal and Cox 2021). However, few attempts have considered the influence of soil geological uncertainty on the prediction of geotechnical properties (Zhang et al. 2021). The

geostatistical approach has the advantage of being able to provide quantitative spatial predictions of soil types (probabilistic geological model) prior to estimating geotechnical properties, while also providing an assessment of spatial uncertainty.

The objective of this paper is to develop a 3D Vs model while considering the uncertainties associated with both geological and geotechnical models. The study was conducted over the city of Saguenay in Eastern Canada, which is a region with highly heterogeneous surficial geology and soil layers of varying thickness and stiffness.. Lithological heterogeneity was characterized through spatial simulation of the main geological units present in the study area (e.g., clay, sand and gravel). The resulting model depicts the probability of occurrence of geological units and their related spatial uncertainties based on the simulation variance. Multivariate statistical analysis was performed to develop the empirical V_s correlations. The geotechnical model was then built by combining the estimated occurrence probabilities of the soil units and the V_s empirical correlations for each soil type. Thus, a consistent spatial distribution of the respective V_s values and their uncertainties were determined in 3D.

3 GEO-MODELING AND CONSIDERED UNCERTAINTIES

Soil variability is primarily rooted in two sources of uncertainty: (1) uncertainty resulting from the inherent variability of the natural process and (2) knowledge-related uncertainties resulting from the statistical inference of a limited number of samples or from measurement imprecisions, i.e., statistical uncertainty or measurement error (Wang et al. 2016). In addition, transformation uncertainty is introduced in the geotechnical variability when field or laboratory measurements are transformed into design soil properties using empirical or other correlation models (Phoon and Kulhawy 1999, Wang et al. 2016). The propagation of the uncertainty to the design soil properties depends primarily on the combination of the analytical methods used and probabilistic analysis

A quantitative geological model obtained by geostatistical simulation is presented, along with the probability of occurrence of the soil types. Probabilities are suggested to describe the different aspects of the uncertainty. The “simulation variance” is introduced as a quantitative measure of geological uncertainty (Yamamoto et al. 2014; Salsabili et al. 2021). Soil units are treated as Bernoulli variables with an outcome of either zero or one, and the variance ($\sigma^2(x_i)$) is computed based on the discrete probability distribution of a random categorical variable (x_i) with an event probability of p_i (Eq. (1) and Figure 1).

$$\sigma^2(x_i) = p_i(1 - p_i), \quad x_i \in \{0,1\}, i \in \{1, \dots, k\} \qquad (1)$$

In the *probabilistic* approach, the mean ($E(Z)$) and combined variance ($\sigma^2(Z)$) of a random geotechnical variable (z_i) with a variance of $\sigma^2(z_i)$ are determined using Eqs. (2) and (3).

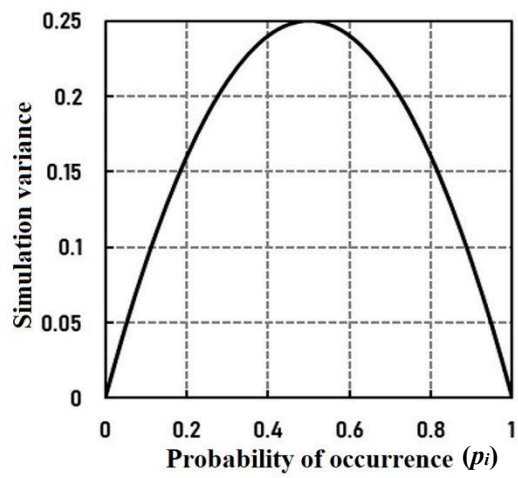

Figure 1. Simulation variance for a Bernoulli variable as a function of the probability of occurrence. When the probability of an outcome is close to 0 or 1, the variance (or uncertainty) is low, whereas when the probability is 0.5, the variance is maximal and equal to 0.25.

$$E(Z) = \sum_{i=1}^{k} p_i \times z_i \ , \qquad (2)$$

$$\sigma^2(Z) = \sum_{i=1}^{k} \left(p_i \times (\sigma^2(z_i) + z_i{}^2)\right) - E(Z)^2 \qquad (3)$$

The uncertainty in V_s is lowest when the simulation variance is zero (i.e., when $p_i = 1.0$) and highest when all members are equally probable (i.e., when $p_i = 0.5$). This approach contributes to a more realistic model of V_s and its associated uncertainties.

4 SAGUENAY CITY STUDY AREA

Saguenay City was selected as the study area due to its relatively high seismic hazard (https://earthquakescanada.nrcan.gc.ca/) and the presence of heterogeneous Quaternary sediments with complex spatial and vertical architecture. It is the largest municipality within the Saguenay–Lac-Saint-Jean region, covering 1136 km² with a population of 147,100. The soil deposits can be grouped into four major categories: till, gravel, clay and sand (Figure 2).

- Till: This glacial sediment is located at the base of the stratigraphic soil column; it is compact and semiconsolidated.
- Gravel: This coarse sediment is mainly of glaciofluvial and alluvial origin; it consists of gravel, sand and occasionally till.
- Clays: These fine postglacial sediments are the most abundant soil type by volume in the study area. Clays are classified as silt, silty clay or clay.
- Sand: This group consists mainly of coarse glaciomarine deltaic and prodeltaic sediments, as well as alluvial sands composed of sand and gravely sand.

Other unconsolidated sediments, such as loose postglacial sediments (alluvium, floodplain sediments, organic sediments, etc.) and landslide colluvium, can also be found in minor proportions. For the purposes of this study, these unconsolidated sediments are classified as sand, clay and/or gravel based on grain size.

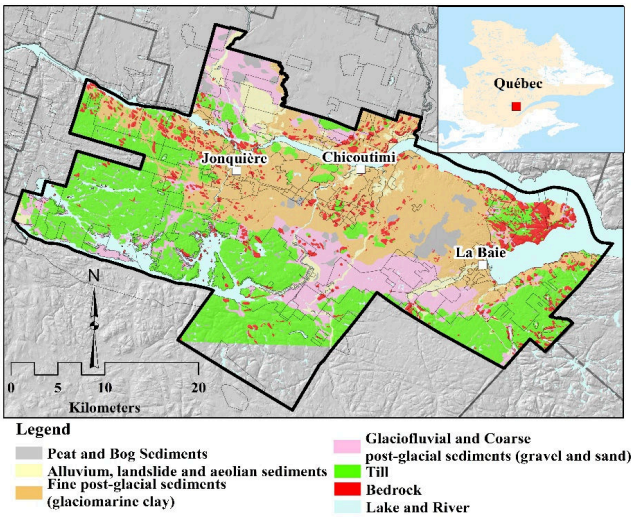

Figure 2. Saguenay city study area: surficial geology map (modified from Daigneault et al. 2011).

5 3D PROBABILISTIC GEOLOGICAL MODEL

Sequential indicator simulation (SIS) was used to determine the spatial boundaries of categorical variables (in this case, clay, sand and gravel) and to develop a model that captures the heterogeneity of soil properties prior to estimating geotechnical parameters (Salsabili et al. 2021). Salsabili et al. (2021) developed the model on the basis of comprehensive datasets, including 3,524 borehole logs, 26 geological cross-sections, and 973 virtual boreholes. They were combined to create the total soil and till thickness maps and to generate the bedrock topography. The space between the top and bottom of each interface was filled with 75 m × 75 m × 2 m blocks to perform the geostatistical simulation. Then, the 3D model of soil type was created by using sequential indicator simulation. Overall, 100 realizations were generated using the conditional SIS method to determine the probability of occurrence (p_i) for each of the postglacial deposits: clay, sand and gravel. The resulting probability values were used to estimate the associated simulation variance (uncertainty). Figure 3 show the probabilistic interpretations of the 100 SIS realizations containing all four surficial soil units.

6 GEOTECHNICAL PARAMETERS

For practical convenience and because the term “geotechnical model” has different meanings in the literature related to stability analysis (Phoon and Tang 2019), the geotechnical model considered in this paper is valid within the limits of elastoplastic behavior before ultimate failure. In this context, the geotechnical model was created similarly to the 3D geologic model in terms of engineering parameters, i.e., V_s.

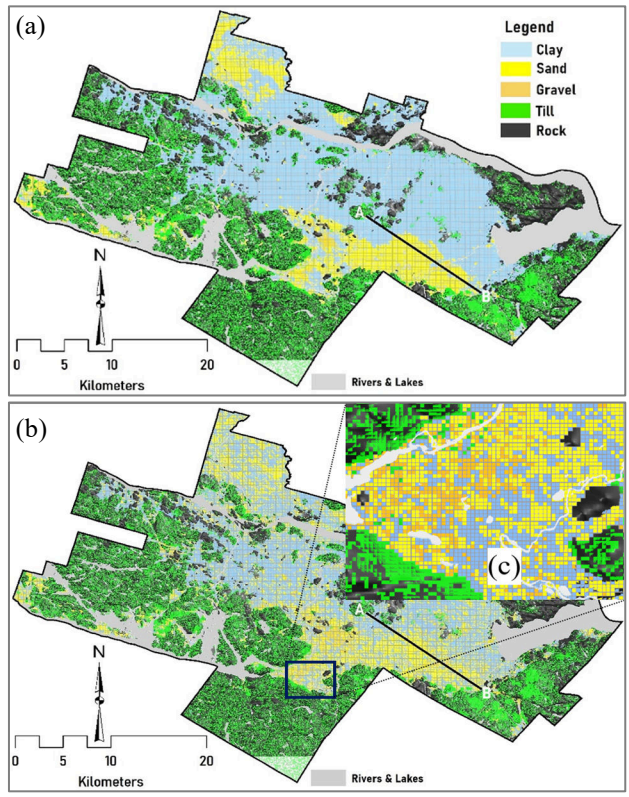

Figure 3. Map of (a) soil units with the highest probability of occurrence at the ground surface and (b) one SIS realization showing sand, clay and gravel. (c) Local blown-up showing the surface soil variability in the SIS map.

The procedure includes two main steps: (I) developing V_s empirical correlations and (II) creating a 3D V_s model that incorporates the probabilistic geologic model and V_s empirical correlations.

6.1 V_s empirical correlations

The seismic piezocone penetration test (SCPTu) is an invasive method that provides optimized V_s intervals and continuous penetration results, allowing the development of reliable empirical correlations between V_s and strength-based soil parameters. For the development of V_s empirical correlations, we 1) perform SCPTu field tests, 2) develop CPTu–V_s correlations by using the results of 15 SCPTu surveys, and 3) estimate V_s on the basis of CPT and SPT data by using empirical correlations for the entire study area. The final step involves developing V_s–depth correlations to assist in determination of the 3D V_s values.

6.1.1 Field testing program

Fifteen SCPTu surveys were carried out using a standard type 2 piezocone. A dual-array seismic cone mounted on the top of the piezocone allows the measurement of arriving vertically propagating seismic body waves. For a given depth, the SCPTu method generates four types of data: V_s, the raw cone tip resistance q_c, the frictional cone resistance f_s and the penetration pore pressure u_2. The field program followed principally the ASTM D5778-12

procedure. In situ tests with invasive methods were conducted during three field campaigns (Figure 4):

- 15 recent SCPTu surveys were conducted by the Université du Québec à Chicoutimi (UQAC) research group. The data include the complete set of q_t, f_s, u_2 and V_s measurements.
- Ninety-one CPT profiles were obtained during the 1980s and 1990s by the Quebec Ministry of Transport (MTQ). The CPT data set is limited to measurements of q_c and f_s. For the purposes of the present study, the field reports were digitalized, and V_s was calculated using the developed sit-specific CPT-V_s correlation.

Sixty-four standard penetration tests (SPTs) were acquired during the 1980s and 1990s by the MTQ. The results were incorporated in the determination of the geotechnical properties of coarse-grained soils.

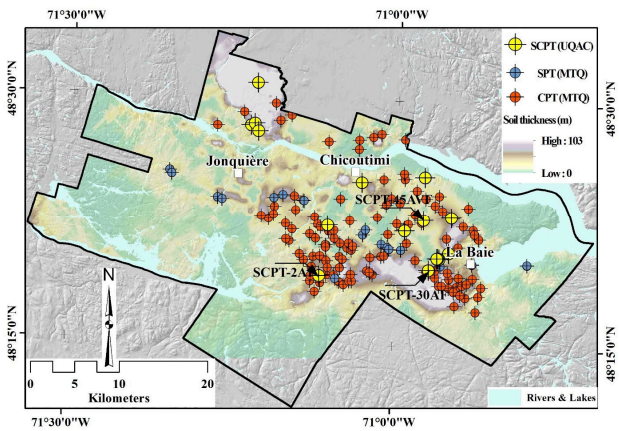

Figure 4. Distribution of geotechnical test sites. The background presents soil thickness (modified from Salsabili et al. (2021), and validation was conducted at the three indicated sites.

6.1.2 Development of CPTu-V_s correlation

The general CPTu–V_s correlation was developed for postglacial soils using 568 data pairs (Eq. (4)). By distinguishing between cohesive (clay-like) and cohesionless (sand-like) soils, simple and robust regression equations for non-piezocone profiles can be developed. The soil behavior type index (I_c) was used to classify soil into two categories: clay ($I_c > 2.6$) and sand ($I_c < 2.6$). The soil-specific CPT-V_s correlations for the clayey soil (Eq. (5)) and for the sandy soil (Eq. (6)) are indicated as follows:

All soils: $V_s = 7.648 q_t^{0.35} I_c^{0.322} D^{0.031} (1 + B_q)^{0.653}$	N = 568	R^2 = 0.692	(4)
Clay: $V_s = 10.052 q_t^{0.379} D^{0.085}$	N = 453	R^2 = 0.813	(5)
Sand: $V_s = 38.757 q_t^{0.174} D^{0.099}$	N = 115	R^2 = 0.545	(6)

where q_t is in kPa; D is depth (m) and B_q is normalized pore pressure (for details on the calculation see Robertson, (2009)).

6.1.3 V_s–depth profile

Following the retrieval and processing of the older MTQ CPT logs, 4600 averaged data pairs of q_t and f_s were generated at 50 cm intervals. The V_s values were predicted by using the developed empirical CPT–V_s correlations (Eqs. (5) and (6)) for sands and clays. In addition, the SPT data were converted into V_s by applying the empirical relationship of Ohta and Goto (1978) for gravel sediments. Then, linear and nonlinear V_s–depth regression analyses were conducted on SCPTu and CPT–V_s data for sand and clay soils (Eqs. (7)– (9)) and on SPT–V_s data for gravels (Eq. (10)). The results are also shown in Figure 5. The standard deviations of the V_s–depth correlations were used as a measure of statistical uncertainty. Note that the data from CPT–Vs and particularly SPT–V_s were subject to epistemic uncertainties. These sources of uncertainty have not been considered in our methodology, due to the limitations in analytical calculations. The use of site-specific V_s correlations for the dominant soil types of the study area (sand and clay) is, however, intended to reduce the epistemic uncertainties.

Sand and Clay mixture: $V_s = 144.9 + 2.55 \times D$	$\sigma_{Vs,SC} = 34\ m/s$	R^2 = 0.43	(7)
Clay: $V_s = 114.5 + 9.4 \times D^{0.76}$	$\sigma_{Vs,clay} = 33\ m/s$	R^2 = 0.59	(8)
Sand: $V_s = 150.47 \times D^{0.149}$	$\sigma_{Vs,sand} = 21\ m/s$	R^2 = 0.66	(9)
Gravel: $V_s = 46.86 + 61.55 \times D^{0.50}$	$\sigma_{Vs,gravel} = 34\ m/s$	R^2 = 0.52	(10)

7 3D GEOTECHNICAL MODEL

A probabilistic method was used to estimate Vs. The V_s values for postglacial deposits were estimated on the basis of the probabilistic approach by using Eq. (2). The V_s values were calculated by using the V_s–depth profiles (Eqs. (8)-(10)) and the probability of soil occurrence (p_i). Then, the associated uncertainty was calculated on the basis of the combined variance approach (Eq. (3)) where the variance of the regression models for each soil type was incorporated for each block. Figure 6 presents the developed 3D geotechnical model, which indicates the spatial distribution of V_s, and its associated uncertainty is shown in Figure 6b. Due to the lack of V_s measurements in glacial deposits and bedrock and the geological similarities between till and crystalline bedrock, the regional V_s values of the glacial deposits and bedrock were calculated from the data obtained by Motazedian et al. (2011) ($V_{s,till}$ = 580 m/s, $\sigma_{Vs,till}$=175 m/s) and Nastev et al. (2016) ($V_{s,rock}$ = 2500 m/s).

To depict the capacity of the proposed method to model the spatial variation of V_s, representative cross-sections are shown in Figure 7. It includes a cross-section of the postglacial soils on top and till and bedrock at the bottom (Figure 7a). In general, the V_s values increase with depth (Figure 7b), but some high anomalies are associated with the soil type (gravel sediments). Figures 7c and 7d present the uncertainty associated with the V_s estimations in two different approaches based on occurrence probability (p_i) of postglacial soil units (geological model). Figure 7c presents the V_s standard deviations (std) in deterministic interpretations of the geological model so it considers only the std of the Vs-depth regressions.

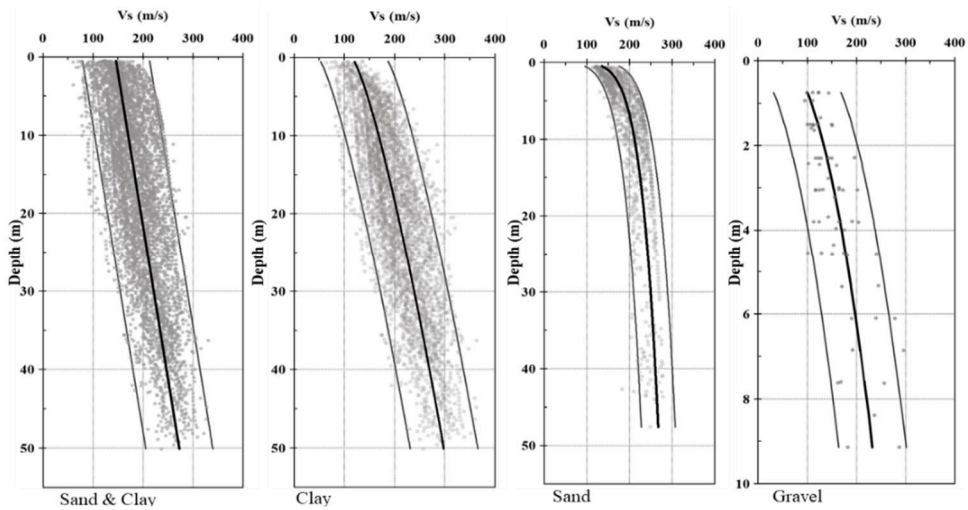

Figure 5. Interval V_s–depth relationships for postglacial sandy and clayey soils. Bold lines indicate average values; gray lines indicate ±2 standard deviations (σ).

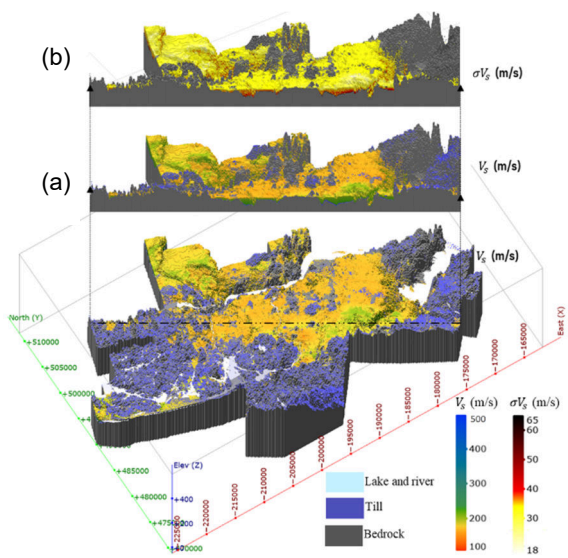

Figure 6. Probabilistic geotechnical model for the city of Saguenay: (a) 3D shear wave velocity and (b) associated V_s standard deviation. The color range indicates the V_s of postglacial deposits. The assumed uniform values for the glacial deposits were $V_{s,till}$ = 580 m/s and $\sigma_{Vs,till}$=175 m/s.

On the other hand, Figure 7d presents the V_s std considering the combined variance of the geological model and V_s-depth regression analysis. We can observe that the uncertainties in the geological model (p_i) have propagated to the V_s values and generally cause higher V_s std in the model. Also, the standard deviations represent the spatial variation of the geological soil units and the predicted V_s data. The efficiency of the developed methodology is depicted by the traces of the geological boreholes. The certainty of the geological model is highest (p_i ~ 1) in the vicinity of the boreholes, and thus, the combined uncertainty of the geological and geotechnical models has its lowest value at these locations. In contrast, as the distance from the boreholes increases, the spatial uncertainty in the prediction of the soil units increases, leading to increased geotechnical model and seismic map uncertainty.

8 CONCLUSION

This study proposed a novel approach for determining the spatial uncertainties of the geological model and propagating these uncertainties to the geotechnical response variable V_s. A probabilistic approach for seismic site characterization was introduced to develop the 3D V_s model and to assess the uncertainty associated with combining various types of uncertainties in building the geological and geotechnical models. The model uncertainty was calculated using the combined variance of the probabilistic geological model and the variance of the V_s–depth regression model.

Given the complex stratigraphic setting and soil type heterogeneity of the study area, sequential indicator simulation was used to predict the probability of occurrence of the postglacial soil deposits. To quantify the uncertainty associated with the geological model, a method for determining the simulation variance was introduced.

Due to the lack of direct V_s measurements, it was necessary to supplement the V_s values inferred from existing CPT logs, which covered most of the study area. SCPT surveys were conducted to develop empirical site-specific CPT-V_s correlations for postglacial sediments in the study area, thereby reducing the epistemic uncertainties associated with the use of existing global correlations.

The V_s correlation functions were developed using nonlinear regression analyses, which incorporated q_t, depth and the SBT indicators for general soil types. In soil-specific correlations, the depth and q_t control the significant variability of V_s, and the developed CPT-V_s correlations were proposed for clay-like and sand-like soils.

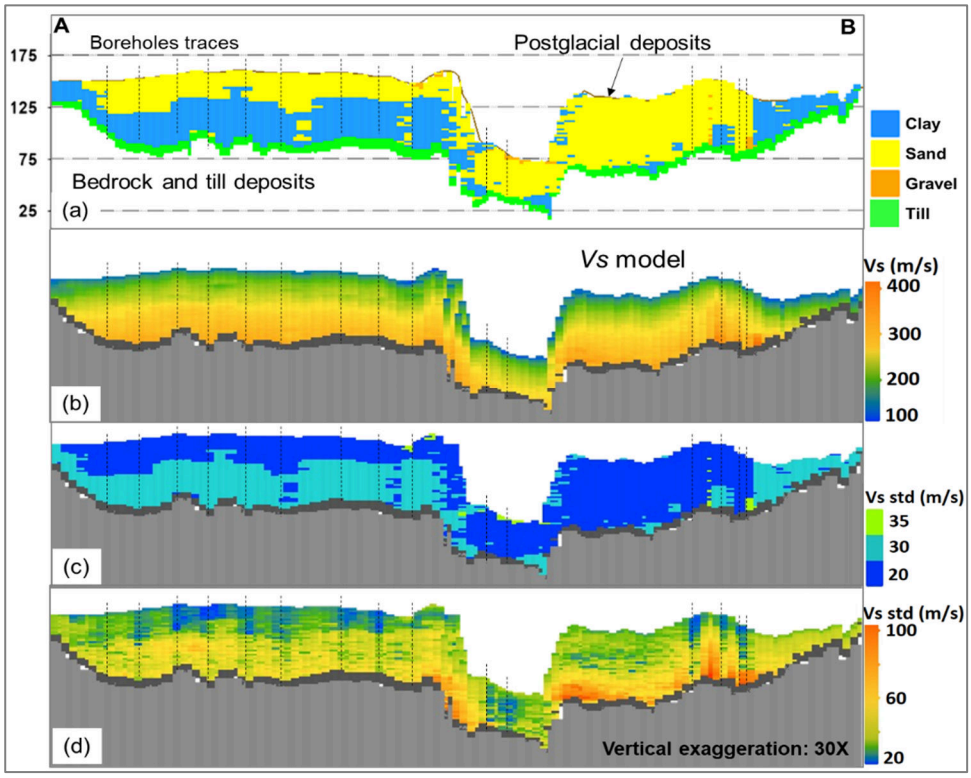

Figure 7. Cross-section of AB presenting the geological boreholes traces and (a) geological stratigraphic (b) V_s spatial distribution, (c) V_s standard deviation obtained by deterministic, and (d) by probabilistic interpretations of the geological model. Dashed vertical lines indicate the borehole traces.

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