Pipeline-Soil Interaction Effects on Vortex-induced Vibration Fatigue Analysis of Multi-Spanning Subsea Pipelines Using Decision Tree Regression

Mobin Abdolalipour & Hodjat Shiri
Department of Civil Engineering – Memorial University of Newfoundland, St. John's, Newfoundland, Canada

GeoCalgary 2022 October 2-5
Reflection on Resources

ABSTRACT

Vortex-induced vibration (VIV) is considered a significant cause of fatigue failure in free-spanned offshore pipelines. The VIV of submarine pipelines involves several physical and structural parameters, where detection of the most influencing parameters for this phenomenon may facilitate the robust VIV response simulation. The pipeline interaction with the seabed soil has a significant impact on VIV-induced fatigue. In the current study, the VIV fatigue performance of multi-spanned subsea pipelines (MSSP) was simulated by incorporation of seabed interaction effects and using Decision Tree Regression (DTR). To this end, initially, the key physical parameters governing the VIV fatigue performance of MSSP were introduced. Subsequently, using these parameters, a set of Decision Tree Regression (DTR), Random Forest Regression (RFR), Extra Tree Regression (ETR), and Support Vector Regression (SVR) models were developed. Ultimately, by analysis of ML models, it's been found that DTR algorithm has excellent performance for modeling VIV fatigue and stress of multispanning pipelines. The obtained result can pave the way for proposing a robust and cost-effective alternative for the initial phases of the pipeline design projects.

RÉSUMÉ

Les vibrations induites par le vortex (VIV) sont considérées comme une cause importante de défaillance par fatigue dans les pipelines offshore à portée libre. Le VIV des pipelines sous-marins implique plusieurs paramètres physiques et structurels, où la détection des paramètres les plus influents pour ce phénomène peut faciliter la simulation robuste de la réponse VIV. L'interaction du pipeline avec le sol du fond marin a un impact significatif sur la fatigue induite par le VIV. Dans la présente étude, les performances de fatigue VIV des pipelines sous-marins à plusieurs travées (MSSP) ont été simulées en incorporant les effets d'interaction du fond marin et en utilisant la régression par arbre de décision (DTR). À cette fin, dans un premier temps, les paramètres physiques clés régissant les performances de fatigue VIV du MSSP ont été introduits. Par la suite, à l'aide de ces paramètres, un ensemble de modèles de régression d'arbre de décision (DTR), de régression de forêt aléatoire (RFR), de régression d'arbre supplémentaire (ETR) et de régression de vecteur de support (SVR) a été développé. En fin de compte, par l'analyse des modèles ML, il a été constaté que l'algorithme DTR a d'excellentes performances pour modéliser la fatigue VIV et le stress des pipelines multi-spanning. Le résultat obtenu peut ouvrir la voie à la proposition d'une alternative robuste et rentable pour les phases initiales des projets de conception de pipelines.

1 INTRODUCTION

Subsea pipelines are a critical element in the transportation of hydrocarbon products. While crossing uneven seabed, these pipelines might be exposed to freespanning conditions, where there is a good chance of vortex shedding vibrations to occur. Vortex-induced vibration (VIV) fatigue damage is one of the most important issues in sub-sea pipeline projects, therefore, the prediction of a pipeline's performance against VIV loads has become an essential topic in offshore engineering. Experimental study of VIV on sub-sea pipelines is expensive; therefore, a detailed numerical simulation of freespanning pipelines can be a logical alternative for experimental studies. In addition, the performance of subsea pipelines against VIV damage is exceptionally non-linear and is dependent on different parameters. Regarding the non-linearity in pipeline's behavior and plenty of parameters influencing fatigue performance, there will be a great number of case studies and the size of the output database will be immense. Regarding the situation, having an industry verified finite element model and employing evolutionary methods for post-processing study cases can be a milestone in understanding the situation and prediction of fatigue performance of freespanning pipelines.

Regarding numerical simulation of freespanning subsea pipelines, a comparison has been made between DNV offered software, known as FATFREE, and newly presented subsea modeling software SPAN2B (Pereira et al. 2008). The comparison process was to validate the performance of SPAN2B for single-span and multispanning pipelines. Since the data presented in their work is the most recent and most detailed data in VIV simulation, their findings can be a good source for the model verification process.

In 2018 evolutionary methods were employed for the VIV modeling of a full-scale bridge(Li et al. 2018). The research was based on long-term real data carried out. The study was about the VIV behavior modeling of a real-world bridge against vortexes caused by wind flow. The input database was filled by sensors installed on the bridge, and the dataset was a collection of data from sensors installed

on the bridge and accumulated for over six years. They employed a decision tree algorithm to classify VIV modes where speed and flow direction of interacting fluid were inputs of the model. The proposed model could identify VIV modes with 94% accuracy. This study was single case research and was conducted on a dataset that is not regeneratable, therefore extended application for their model is impossible.

Classical machine learning methods have been employed for modeling dynamic fluid loadings (Peeters et al. 2020). They have developed a mapping method to relate wake variables to fluid loading. Their model employed machine learning to map the output velocity field to the transverse force coefficient on a submerged circular cylinder. The accuracy of the model was 95%. The trained neural network was simple, and the study proved that even simple neural networks can be useful in VIV problems but like the previous study, the studies were performed on a very specific case study and there is no guarantee that the developed model can be useful for general VIV problems. Machine learning and specifically genetic algorithm can be employed for system identification of VIV-involved systems (Ooi et al. 2020). In 2020, an iterative algorithm has been developed by using machine learning to find a reduced dimension for physical parameters affecting VIV. The results propose a VIV parametric model for a pipe with Helical Strakes where the model is developed by using the improved recursive least squares method. Input data for this method were not from practical measurements and all were artificially generated (Ma et al. 2020). Output results had been validated by a genetic algorithm. The goal of this study was to reach a minimum amplitude of VIV. In 2021 a method has been developed for the classification of vortex shedding modes of bladeless wind turbines (Cann et al. 2021).

The research was based on practical data from measurements on small and local wind turbines and derived data from conducted simulations. Machine learning was deployed on data output from simulations.

Machine learning techniques played a vital role in the development of the final model which could successfully classify vortex shedding modes produced by the oscillating cylinder of a bladeless wind turbine.

Evolutionary methods can be useful in the active control of pipeline under VIV loads and ocean energy harvesting (Mei et al. 2021) evolutionary methods for active control of a cylinder under VIV loads was successful. The method was to actively control the oscillation by a jet flow which is controlled by data from an artificial neural network. A control algorithm based on deep reinforcement learning was used to train a artificial neural network, whereas OpenFoam was utilized for modeling computational fluid dynamics (CFD).

Application of machine learning in VIV extended for numerical simulation of two parallel pipes by applying lattice Boltzmann method for computational fluid dynamic modeling (Gu et al. 2021). The main goal of the research was to predict the amplitudes upstream and downstream by machine learning (ML) algorithms. Firstly, they have extracted four main parameters affecting amplitudes of the upstream cylinder and downstream cylinder and used them for training two machine learning models. Secondly, three ML techniques namely decision tree regressor (DTR), gradient boosting regressor tree (GBRT), and random forest (RF) techniques have been evaluated to reveal the most accurate method for this study. Results show that the GBRT method has the most capability to predict amplitudes of upstream pipe and downstream pipe.

In the field of turbulent flow studies, in 2021, a solution has been proposed in which a neural network is trained by data from parameterized Navier-Stokes equations. The simulations consist of a turbulent flow with Reynolds number 104 passed through a cylinder under VIV loads(Bai and Zhang 2022).

Presented research aims to investigate the performance of classic machine learning regression methods for the prediction of VIV fatigue performance of multispanning pipelines. In this study, a parametric study is performed on an FEA model of a subsea freespanning pipeline. A dataset of 14800 different conditions of seabed and pipe characteristics has been prepared. Afterward, four different machine learning algorithms including DTR, ETR, RFR, and SVR selected to train ML models for the prediction of VIV stresses in inline, crossflow, and maximum inline and crossflow stresses. Finally, based on the best model, the most influential parameters affecting VIV fatigue performance were selected.

2 FINITE ELEMENT MODEL

In this research, ABAQUS software was carried out for simulation of the pipeline's behavior under freespanning conditions. Soil is modeled as sets of springs in 3 dimensions based on the Winkler foundation. Values of spring stiffness are drawn from DNV RP-F105 (DET NORSKE VERITAS 2006)equal to characteristics of medium sand seabed.

Stresses in inline and crossflow directions have been calculated based on DNV RP-F105. Soil stiffness has been excluded from parametric studies, therefore exact compliance between verified simulation and case studies will remain intact. Soil stiffness parameters can be achieved based on Equation 1 and Equation 2.

Parameters K_V, K_L, and K_A are equivalent spring stiffness for vertical, lateral, and axial directions, respectively:

$$K_V = \frac{C_v}{1-v}\left(\frac{2}{3}\frac{\rho_s}{\rho} + \frac{1}{3}\right)\sqrt{D} \quad [1]$$

$$K_L = K_A = C_L(1+v)\left(\frac{2}{3}\frac{\rho_s}{\rho} + \frac{1}{3}\right)\sqrt{D} \quad [2]$$

Where D is the outer diameter of the pipe, $\frac{\rho_s}{\rho}$ is the relative density of steel pipe to ambient water. Coefficients C_v and C_L are selected from. Soil parameters have been presented in Table one.

Table 1. Dynamic stiffness factor and static stiffness for pipe-soil interaction in sand

Sand type	C_v $\left(\frac{kN}{m^{\frac{5}{2}}}\right)$	C_L $\left(\frac{kN}{m^{\frac{5}{2}}}\right)$	$K_{V,S}$ $\left(\frac{kN}{\frac{m}{m}}\right)$
Loose	10500	9000	250
Medium	14500	12500	530
Dense	21000	18000	1350

Figure 1 presents a schematic of seabed topography. Modeled multispanning condition consists of 2 consecutive single spans which are separated by a shoulder bump. It is assumed that the shoulder and seabed are at the same level and gap depth remains constant through the length of span one and span two.

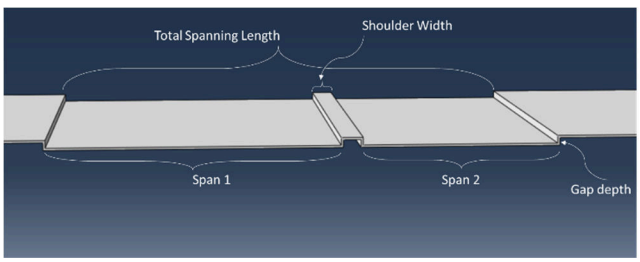


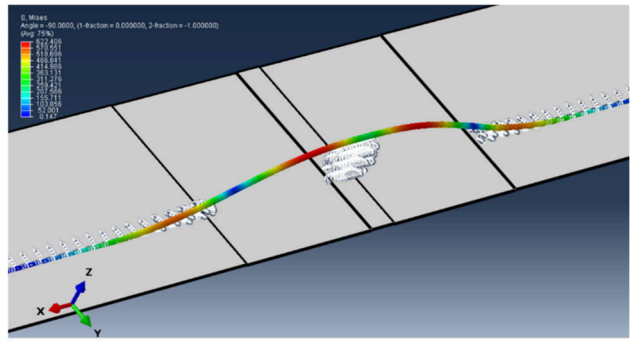
Figure 1. Seabed topography of multispanning pipelines.

Figure 2 and Figure 3 demonstrate exemplary mode shapes of pipelines in crossflow and inline directions. Detailed information about the model of exemplary mode shapes is listed in Table 2.

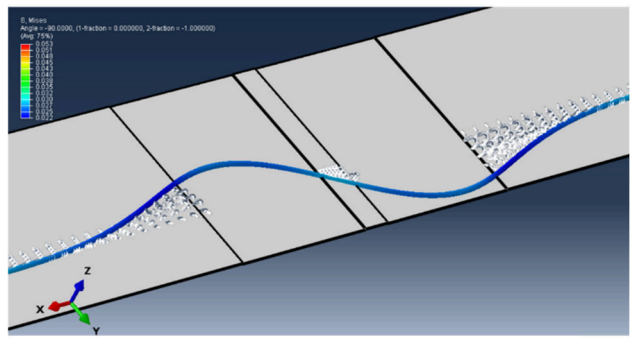
Since gap depth and pipe depth are relatively much smaller than span length, the pipeline is rendered to display a larger diameter than the actual value and pipeline deformations within natural mode shapes are up scaled for better visual understanding.

Table 2.Sample multispanning model information.

Parameter	Value	Unit
Soil type	Medium sand	-
Shoulder width	4x pipe diameter	-
Total spanning length	60x pipe diameter	-
Minimum span length	0.40x total span length	
Gap depth	1.5x pipe diameter	
Pipe diameter	8.625	inch



(a)



(b)

Figure 2. Sample mode shape of freespanning pipeline (a) Crossflow (b) Inline.

2.1 Model Validation

Seabed soil properties meet the class of Medium-Sand based on DNV RP-F105. In order to verify the FEA model, a comparison was performed between the simulated model and the results of DNV FATFREE and SPAN2B software presented by A.Pereira.

A comparison was conducted based on values of first natural frequencies in inline and crossflow directions between the developed model and results of DNV FATFREE and SPAN2B software. The model is a single-spanned pipeline with properties mentioned in Table 1.

Table 3. Characteristics of validation model.

Parameter	Value	Unit
Pipe outer diameter	12.75	inch
Pipe wall thickness	0.875	inch
Soil type	Medium Sand	-
Freespanning length[1]	60 up to 240	-

[1]Ratio of freespanning length over pipe outer diameter.

First modes of natural frequencies both in in-line and crossflow directions have been considered as a comparison subject. The simulated model showed a similar trend of response to increase in span length with DNV FATFREE and SPAN2B.

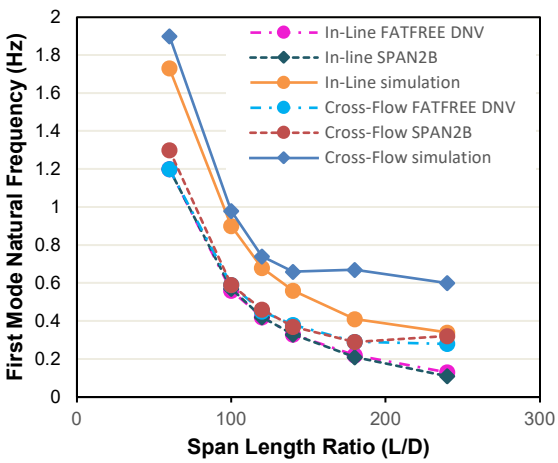


Figure 3: First mode natural frequency comparison.

Based on the information presented in Figure 3, the developed FEA model shows similar behavior to software SPAN2B and DNV FATFREE. Trends of dropping values of first natural frequencies in Inline and Crossflow direction follow a similar pattern to SPAN2B and DNV FATFREE, therefore, the model is trustable, and we can perform the parametric study.

2.2 Parametric Study

To achieve a reliable prediction model by using machine learning methods, an abundant number of different cases is necessary. To attain a reliable ML dataset, a thorough parametric study was performed. There are seven varying parameters in the ML models, we can assume them as model features. Most model features are reformed with other parameters to make the final feature a dimensionless parameter.

The physical and mechanical properties of pipes are selected from standard engineering pipes. Pipe schedule XS is the selected reference for the pipe's physical dimensions. Pipe mechanical properties are assumed to be identical to industrial steel which is used in the offshore industry.

Pipe and soil properties have been chosen from standard pipes and approved DNV(DET NORSKE VERITAS 2021) reports respectively. The final dataset has a total of 14800 entries, where every entry has a distinct set of values for attributed features. After the preparation of output files by ABAQUS, post-processing calculations were carried out by using python script, and finally, inline, crossflow, and maximum of Inline and crossflow mode shapes were calculated and considered as target values. Therefore, we have three targets. Target values for stresses for inline and crossflow directions can be calculated based on DNV RP-F105.

$$Target1 = S_{IL} = 2.A_{IL}.\left(\frac{A_Y}{D}\right).\psi_{\alpha,IL}.\gamma_s \quad [3]$$

$$Target2 = S_{CF} = 2.A_{CF}.\left(\frac{A_Z}{D}\right).R_k.\gamma_s \quad [4]$$

$$Target3 = \max\{Target1, Target2\} \quad [5]$$

Where S_{IL} and S_{CF} are VIV stress in inline and crossflow directions respectively. A_{IL} and A_{CF} are unit stress amplitude in respective directions. The $\frac{A_Z}{D}$ and $\frac{A_Y}{D}$ are also VIV amplitudes in crossflow and inline direction which can be calculated by methods prescribed in DNV RP-F105. We also need to check for the occurrence of crossflow induced inline vibration, which is called the figure 8 condition, and if necessary, update inline stresses. $\psi_{\alpha,IL}$ is the reduction factor for competing inline modes, γ_s is the safety factor for stress amplitudes, and R_k is the amplitude reduction factor due to damping.

Inline and crossflow VIV amplitudes are presented in Figure 4 and Figure 5.

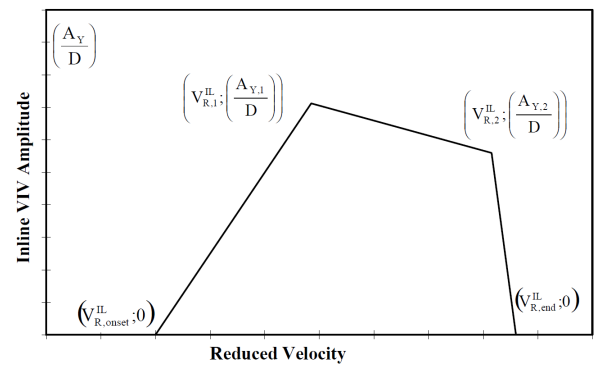


Figure 4. Inline VIV amplitude(DET NORSKE VERITAS 2006).

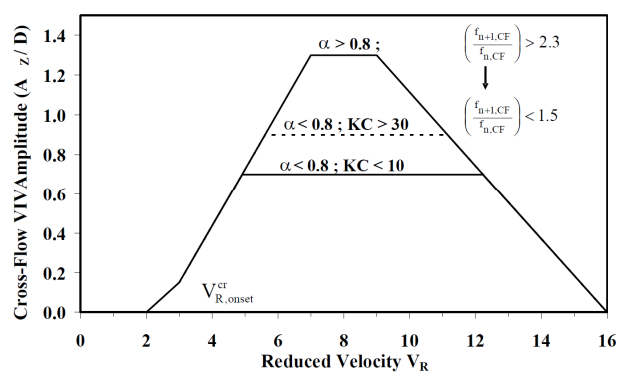


Figure 5. Crossflow VIV amplitude (DET NORSKE VERITAS 2006).

3 MACHINE LEARNING METHODS

Four different ML methods have been considered for regression between input features and output targets. Implemented methods are Decision Tree regression, Random Forest regression, SVM regression, and Extra Tree regression(Rishal Hurbans 2020).

In the Decision Tree Regression model (DTR), the model divides the dataset into smaller subsets and repeats

this process until there is enough homogeneity in the final subsets. Statistical parameters can interpret the performance of ML models as R^2 score, root mean square error, and mean absolute error between predicted values and real values.

The random forest regression algorithm (RFR) is also based on regression trees, but despite the decision tree algorithm, in random forest, multiple trees are developed and the tree with the best performance is chosen as the final model. In RFR, in addition to given data, bootstrap replicas are also generated and used in decision tree developments. Bootstrap replicas are data that are generated based on given dataset to fill missed data or to improve the quality of decision tree.

Extra trees regression algorithm (ETR) is primarily similar to RFR, but despite RFR, in ETR, only the whole given dataset is used for developing trees, and there is no bootstrap replica. Like RFR, in ETR, the model with the best performance will be chosen as the final model.

Support vector regression machine (SVM) is an ML algorithm which can be useful for both classification and regression (SVR), in this method, data of a dataset with m features is plotted in a m-dimensional space where each entry is a datapoint. In this space the value of each feature is the value of a particular coordinate. Then, the goal is to find the hyperplane that best differentiates datapoints into two subsets. This algorithm can be used for both classification and regression problems.

3.1 Model Features

The dataset for this study has 6 features. Except for current velocity and pipe diameter, all other features have been defined as dimensionless parameters. Definition of these features are as follows:

- D represents pipe diameter, overall, 11 different sizes for pipe are considered. The values have been extracted from engineering references from the category of pipe schedule XS which is suitable for heavy-duty operations. Applicable pipe sizes range from size 5 to 24 based on a list of standard pipe sizes.
- Parameter Relative wall thickness is defined as the inverse of the respective pipe's wall thickness, t, to the pipe's diameter D. The value can be attained by the ratio $\frac{D}{t}$. The pipe's wall thickness is assumed to impact the pipe's strength and deflection.
- Gap depth is a dimensionless variable expressing the ratio of the depth of freespanning, e, to pipe's diameter D. This feature can be represented as $\frac{e}{D}$.
- The feature freespanning length or total freespanning length is the ratio between the length of total multispanning length as shown in Figure 1 to pipe diameter D.
- Span 1 is the ratio of the span length 1 to the total freespanning length.
- Current is defined as current flow velocity; this parameter plays a vital role in calculating reduced velocity. Current velocity ranges from 0.1 m/s up to 4m/s.
- Feature shoulder width can be calculated as a ratio of the width of the middle support shoulder to the pipe diameter. This feature is dimensionless. Larger shoulder width can support the pipeline and decrease vibration amplitude because of the soil stiffness effect and a narrow shoulder may trigger a high concentrated load on shoulder edges.

Table 4. Dataset features.

Features	Value	Unit
D	Sizes 5 to 24 SCH XS	m
Relative wall thickness	Sizes 5 to 24 SCH XS	-
Pipe schedule	XS	-
Gap depth	0.5 - 3 x pipe diameter	-
Freespanning length[1]	60 up to 240	-
Shoulder width	1 – 5 x pipe diameter	
Span 1 ratio[2]	0.1-0.5	-
Current	0.1-4	m/s

1 Total freespanning length including span 1, span 2, and shoulder.
2 Ratio of length span1 to total freespanning length.

A linear correlation between all features and targets is shown in Figure 6. This correlation presents a degree of linear relativity between every two sets of values, including features, targets, or a feature and a target. This correlation is just linear and can give an overall perspective on the dependency of features and targets. This parameter is dimensionless and is defined as the fraction of 1. Number 1 represents 100% relativity which is only possible if we compare a data with itself, number -1 indicates that two parameters are negatively related, meaning that an increase of parameters happens when the other parameter is decreased. Number 0 indicates that two parameters are linearly independent.

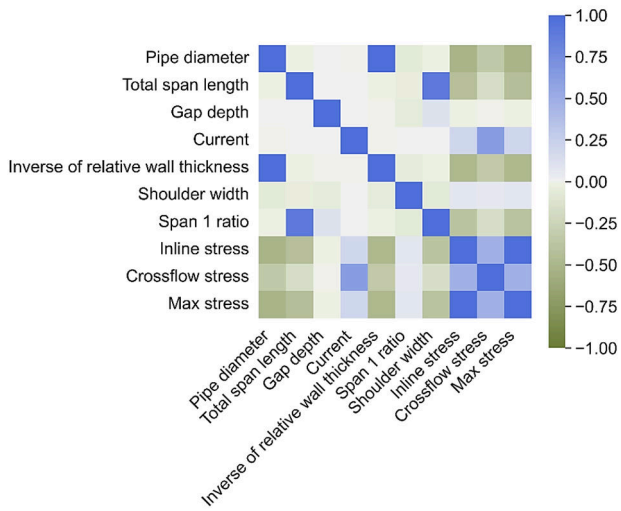


Figure 6. Linear correlation between all features and targets.

3.2 Model Performance Evaluation

In this research, four different machine learning methods have been utilized including decision tree regression, random forest regression, extra tree regression, and support vector machine regression. In the first step, the dataset is split into training and test divisions, and then all individual models have been trained for every set of targets and learning methods.

The performance of each method on targets has been studied individually and method performance evaluated by measuring performance indicators including the R^2 score, mean absolute error, and square root mean error. R^2 score is amount of variance in the predictions by the dataset, this parameter is explained in Equation 6:

$$R^2 = 1 - \frac{\Sigma\left(y_i - y_{i_p}\right)}{\Sigma(y_i - \bar{y})} \quad [6]$$

Where y_{i_p} is regression outcome. For mean absolute error we have:

$$MAE = \frac{\Sigma\left(y_i - y_{i_p}\right)}{n} \quad [7]$$

Where, n is the total number of entries (data points). And for mean square error we have:

$$RMSE = \sqrt{\frac{\Sigma\left(y_i - y_{i_p}\right)^2}{n}} \quad [8]$$

3.3 Machine Learning Model Implementation

For a true evaluate the performance of machine learning models, all the models have been trained with identical sets of training data. To achieve better performance, values of features have been scaled into similar data ranges. In this study, all the features have only positive values, therefore they all have been linearly scaled into values between 0 and 1.

Despite features, target parameters have not been scaled for models of DTR, ETR, and RFR, but target values were so imbalanced for the SVR model properly, therefore, targets have been transformed into highly balanced data ranged by the BOXCOX method and after training SVR model, a grid-search performed on trained SVR model to obtain best parameters regarding the data.

After tuning, the SVR performance improved. Prior to the evaluation of the SVR model, predicted values for the target have been inversely transformed from the BOXCOX to the original range and then performance indicators for the SVR model were calculated. The formula of the BOXCOX transform is presented in Equation 9, where λ can be identified by try and error, built-in function for boxcox in python libraries can predict the value of λ.

$$y(\lambda) = \begin{cases} \frac{y^{\lambda}-1}{\lambda} & if\ \lambda \neq 0; \\ \log y & if\ \lambda = 0 \end{cases} \quad [9]$$

DTR model had excellent performance on all of the targets. The results from the SVR model were not satisfying, therefore, using the grid-search technique, a hyper parameter tuning was performed on the SVR model to improve model accuracy. RFR and ETR models showed good performance, but overall, DTR was better than others.

Table 5. Performance of ML methods on Inline stress prediction.

Model	R2 Score	MAE	RMSE
DTR	0.99	2.91	33.4
ETR	0.99	4.33	42.5
RFR	0.99	5.85	38.2
SVR	0.93	45.3	134

Table 6. Performance of ML methods on Crossflow stress prediction.

Model	R2 Score	MAE	RMSE
DTR	0.99	6.33	51.5
ETR	0.99	9.03	41.3
RFR	0.99	6.33	51.5
SVR	0.93	340	832

Table 7. Performance of ML methods on Maximum stress prediction.

Model	R2 Score	MAE	RMSE
DTR	0.99	5.97	41.8
ETR	0.99	7.88	30.8
RFR	0.99	12.9	42.9
SVR	0.96	227	608

After analyzing the performance of each ML model, the best models have been chosen for extraction of feature influence on targets. Figure 7, Figure 8, and Figure 9 display the influence of each parameter on inline stress, crossflow stress, and maximum stress respectively. The most important parameters for each stress are listed in Table 8.

Table 8. Most Influential parameters on Fatigue stress.

Stress	Most important parameter
Inline stress	Current velocity
Crossflow stress	Total freespanning length
Maximum stress	Total freespanning length

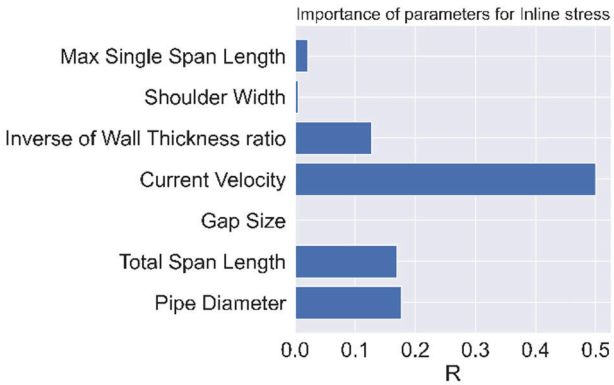


Figure 7. Importance of parameters on Inline stress.

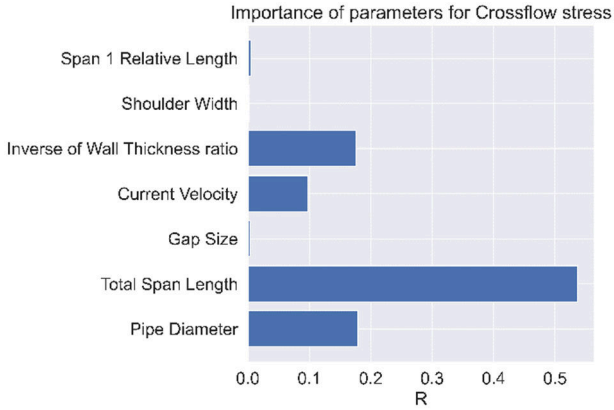


Figure 8. Importance of parameters for Crossflow stress.

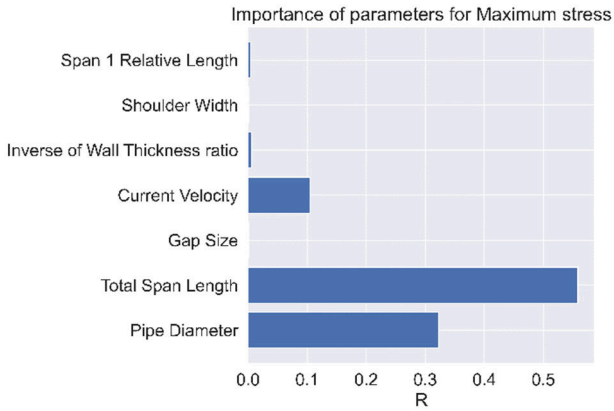


Figure 9. Importance of parameters for Maximum stress.

4 CONCLUSION

This research was conducted to evaluate the performance of machine learning algorithms on VIV fatigue life prediction of multispanning pipelines. VIV stresses are key parameters affecting VIV fatigue life expectancy of multispanning pipelines. In this research, a thorough, parametric study was performed on a finite element model developed in ABAQUS software. The post-processing step was performed based on DNV RP-F105 and the final dataset has been used for training different machine learning models.

It has been discovered that the DTR model has excellent performance for modeling and predicting VIV stresses. The results indicate that total freespanning length in a multispanning pipeline is the key parameter for VIV stresses which lead to VIV. For inline VIV stress, current velocity is the most critical parameter, which positively affects the inline stress, meaning that higher current velocity will lead to larger inline VIV stress. For crossflow VIV stresses, total freespanning length is the most important parameter. And regarding the maximum stresses, overall, crossflow stress is generally larger than inline stress and governs the maximum stress value.

5 ACKNOWLEDGEMENT

The authors gratefully acknowledge the financial support of the “Wood” through establish-ing Research Chair program in Arctic and Harsh Environment Engineering at the Memorial University of Newfoundland, the “Natural Science and Engineering Research Council of Canada (NSERC)” and the “Newfoundland Research and Development Corporation (RDC) (now InnovateNL) through “Collaborative Research and Developments Grants (CRD)”. Special thanks are extended to Memorial University for providing excellent resources for conducting this research program.

6 REFERENCES

Bai, X.-D., and Zhang, W. 2022. Machine learning for vortex induced vibration in turbulent flow. Computers & Fluids, 235: 105266. doi:10.1016/j.compfluid.2021.105266.

Cann, M., McConkey, R., Lien, F.-S., Melek, W., and Yee, E. 2021. Mode classification for vortex shedding from an oscillating wind turbine using machine learning. Journal of Physics: Conference Series, 2141(1): 012009. doi:10.1088/1742-6596/2141/1/012009.

DET NORSKE VERITAS. 2006. DNV-RP-F105 FREESPANNING PIPELINES RECOMMENDED PRACTICE. DNV.

DET NORSKE VERITAS. 2021. DNV-RP-F105 FREESPANNING PIPELINES.

Gu, S., Wang, J., Hu, G., Lin, P., Zhang, C., Tang, L., and Xu, F. 2021. Prediction of wind-induced vibrations of twin circular cylinders based on machine learning. Ocean Engineering, 239: 109868. doi:10.1016/j.oceaneng.2021.109868.

Li, S., Laima, S., and Li, H. 2018. Data-driven modeling of vortex-induced vibration of a long-span suspension bridge using decision tree learning and support vector regression. Journal of Wind Engineering and Industrial Aerodynamics, 172: 196–211. doi:10.1016/j.jweia.2017.10.022.

Ma, L., Resvanis, T.L., and Vandiver, J.K. 2020. Using Machine Learning to Identify Important Parameters for Flow-Induced Vibration. *In* Volume 4: Pipelines, Risers, and Subsea Systems. American Society of Mechanical Engineers.

Mei, Y.-F., Zheng, C., Aubry, N., Li, M.-G., Wu, W.-T., and Liu, X. 2021. Active control for enhancing vortex induced vibration of a circular cylinder based on deep reinforcement learning. Physics of Fluids, 33(10): 103604. doi:10.1063/5.0063988.

Ooi, C.S., Lim, M.H., Lee, K.Q., Kang, H.S., and Leong, M.S. 2020. System Identification of Helical Strakes Suppressed Vortex-induced Vibration for Flexible Pipes. Pertanika Journal of Science & Technology, 28(2).

Pereira, A., Bomfimsilva, C., Franco, L., Tardelli, L., and Eigbe, U. 2008. In-Place Free Span Assessment Using Finite Element Analysis. *In* Volume 3: Pipeline and Riser Technology; Ocean Space Utilization. ASMEDC. pp. 191–196.

Rishal Hurbans. 2020. grokking artificial intelligence algorithms.