Multimodal Search for Critical Slip Surfaces in Slopes Reinforced with Piles

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ABSTRACT
Piles can be employed to increase the available restoring force, and subsequently the factor of safety of sliding in slopes. In this paper, a 3D model of a slope exhibiting multiple failure modes with factor of safety near unity is reinforced using pile elements. The results of the analysis with piles are obtained using a new coupled approach which combines the results of finite element analysis for each pile with the iterative solution of limit equilibrium (LE) analysis. This coupled procedure is combined further with the novel Multiple Minimum Optimization (MMO) metaheuristic method which adapts the Particle Swarm Optimization (PSO) method to identify multiple local minima of the factor of safety. The results of the analysis are also compared with 2D sections containing the piles analyzed using 2D LE analysis. The results of the 2D and 3D couple analyses for the slope were found to be in good agreement.

RÉSUMÉ
Les pieux peuvent être utilisés pour augmenter les forces de stabilisation, et par conséquent, le facteur de sécurité de glissement dans les pentes. Dans cet article, un modèle 3D d'une pente présentant plusieurs modes de rupture avec un facteur de sécurité proche de l'unité est renforcé à l'aide d'éléments de pieux. Les résultats de l'analyse avec pieux sont obtenus à l'aide d'une nouvelle approche couplée qui combine les résultats de l'analyse par éléments finis pour chaque pieu avec la solution itérative de aa méthode d'Equilibre Limite (EL). Cette procédure couplée est en outre combinée avec la nouvelle méthode métaheuristique d'optimisation multimodal (OMM) qui adapte la méthode d'optimisation d'essaim de particules (PSO) pour identifier plusieurs minima locaux du facteur de sécurité. Les résultats de l'analyse sont également comparés aux sections 2D contenant les pieux analysés à l'aide de l'analyse EL 2D. Les résultats des analyses de couple 2D et 3D pour la pente se sont avérés en bon accord.

1 INTRODUCTION

Traditional Limit Equilibrium analysis provide the critical slip surface and its corresponding factor of safety (FS). Because of the simplicity of LEM approach to be combined with metaheuristic approaches to search for the critical slip surface and its corresponding factor of safety, this method is highly popular for the slope stability analyses (Cheng et al. 2007; Cheng et al. 2012; Zhao et al. 2008; Tian et al. 2009; Li et al. 2009; Chen and Chen 2011; Kalatehjari et al. 2012, 2014; Fisher and Goodale 2021; Javankhoshdel et al. 2021; Su and Shao 2021). Among the various metaheuristic approaches, Particle Swarm Optimization technique (PSO) (Kennedy and Eberhart 1995) is one of the most popular. However, focusing the search effort on locating the single critical failure surface has drawbacks. As pointed out by Reale et al. (2015) little research has been completed on slopes which could develop several critical slip surfaces with similar FS values. There are cases where the global minimum is of little practical importance, e.g. when a slope is susceptible to multiple failure mechanisms, e.g. slopes with multiple benches and/or layers (Javankhoshdel et al. 2021).
Recently, a method has been developed that has the ability of finding multiple minimum failure surfaces and their corresponding FS values - multimodal optimization (MMO) (Li et al. 2020). In this approach. a multimodal particle swarm algorithm, LIPS-R (Li et al. 2020), is used to find multiple local minima, or critical failure surfaces.

Piles are often used to reinforce slopes as they increase the available restoring force when calculating the factor of safety against sliding. In LEM, a restoring force is applied on the slipping surface where it intersects a pile element. Slopes that do not meet minimum safety factor requirements for overall stability stipulated in applicable codes and standards can be reinforced with piles to increase the factor of safety. However, if multiple failure modes exist in a slope, it is important to ensure that all the potential failure modes satisfy the minimum requirements instead of just the critical slip surface. The use of MMO is therefore useful to more efficiently identify and evaluate the factor of safety in multiple regions of a slope in a single analysis.

Recently, a coupled approach has been introduced which combines the results of finite element analysis of the displacement of soil along the pile with the traditional limit equilibrium iterative formulation. It combines the formulations for axial (Rocscience 2018) and lateral (Reese & Van Impe 2011) displacements to the piles to determine the restoring reaction of the pile onto the slip surfaces, depending on direction and the location and of the slip surface where it intersects the pile. This paper showcases the extension of the coupled analysis method for piles in LEM into 3D via the Slide3 and RSPile (Rocscience 2021) software and comparison between the results of 2D and 3D analyses.

In this study, a slope is reinforced with piles, and then the MMO factor of safety analyses are conducted with considering the coupled 2D and 3D LEM slope stability analysis and FEM pile analysis.

2 OVERVIEW OF THE MMO LEM METHOD

Particle Swarm Optimization (PSO) is a global optimization method which simulates the swarming behavior of various species in the real world to search for the global minimum value and location of an objective function. In PSO, particles are spawned which move through the solution space to search for the global minimum. It has been applied in unimodal problems in a wide array of disciplines. With respect to LEM, PSO has been utilized to find the critical slip surface corresponding to a minimum factor of safety in a slope. However, for many real-world problems it is often desirable to find several local optima rather than just a single global minimum. As such, the PSO method, among other population-based niching methods, has been extended to solve multi-modal optimization (MMO) problems (Li et al. 2017).

Qu et al. (2013) proposed a locally-informed particle swarm (LIPS) method wherein the particles store limited information about the best positions only within their local vicinities. This allows better exploration of the solution space around local minimums in MMO problems.

Li et al. (2020) introduced a LIPS-R algorithm, which is based on the LIPS method but includes an additional radius filter. The radius filter discards solutions that are too close to each other in either solution or physical space. It works by first sorting the particles in the final iteration of LIPS by order of fitness value (in this case, the factor of safety). In ascending order of fitness value, the positions of the corresponding particles are assessed to determine if they are too close to another particle with lower FS. The assessment is based on a cutoff distance corresponding to a percentage of the model space, *r*. Two surfaces are too similar to each other if the respective faces on the bounding boxes are within *r* distance of each other. In such a case, the surface with higher factor of safety is discarded. After filtering, LIPS-R outputs all the optima that are at least *r* apart from each other. In this study *r* equal to 10% of the solution space was found to perform well. This suggests the optima will be at least 10% of the search space away from the others.

3 LEM SLOPE STABILITY COUPLED WITH FEM PILE ANALYSIS

In LEM, support elements which intersect the slip surface are considered to provide a restoring force onto the slipping mass. The magnitudes and directions of the forces can be based on the specifications and assumptions of the designer and can either be as simple as constant values or as complex as requiring coupled analyses. Commonly, the directions of the forces are assumed to be either tangent to the slip surface, perpendicular to the pile, horizontal, or some combination of the above. In more complicated analyses, the assumed magnitudes and directions of the support forces can be related to both the mechanical deformation of the support and its surrounding soil.

In this study, a coupled analysis method is introduced whereby the finite element method (FEM) is used to determine the mechanical behavior of piles used as supporting elements in a slipping mass. The coupled procedure involves determining the displacement of the surrounding soil, which interacts as an elastic foundation against pile deformations resulting from the forces from the slipping mass.

The displacement of the surrounding soil is first assumed based on the slipping direction and intersection location of the slipping surface with the pile. The soil displacement is applied as an initial condition in the FEM analysis for the pile. Knowing that it is difficult to predict the actual displacement of the soil around the pile during slippage, the magnitude and direction of the soil displacement is typically assumed to be a uniform vector above the point of intersection of the slip surface with the pile. The soil displacement is then partitioned along the three principal axes of the pile – that is, the major and minor lateral axes and longitudinal (axial) axis.

Once all the initial conditions are set up, the FEM model for the pile is solved iteratively. Note that by this method, the solutions on the three principal axes are assumed to be independent. Reese & Van Impe (2011) proposed a formulation for lateral deformations of the pile, which is effectively summarized by Rocscience (2022), respectively. Rocscience (2018) proposed an analogous finite element method for determining the axial deformations of an axially loaded pile. The governing differential equations for the axial and lateral deformations of the pile are given in Eq. 1 (Rocscience 2018) and Eq. 2 (Reese & Van Impe 2011), respectively.

$$E_p A_p \frac{d^2u}{dx^2} + \tau \cdot C_p = 0 \qquad [1]$$

$$E_p I_p \frac{d^4y}{dx^4} + P_x \frac{d^2y}{dx^2} + E_{py} y - W = 0 \qquad [2]$$

Where *u* is the axial displacement and *y* is the lateral displacement along distance *x* along the pile. E_p is the elastic modulus of the pile, A_p is the cross-sectional area of the pile, I_p is the moment of inertia of the pile corresponding to the axis for which the lateral analysis is being performed, and C_p is the circumference of the pile segment at depth *z*. Along some distance *x* along the pile, τ is the soil unit skin friction, and E_{py} is the soil reaction modulus based on the magnitude of the distributed load *W*. Finally, P_x is the axial load on the pile head (if applicable).

The stress-strain relationship between the pile and soil can be expressed using axial stress-strain (*t-z* curves) curves and lateral stress-strain (*p-y* curves) curves. *Some t-z* curves are available from the American Petroleum Institute (2002) and some *p-y* relationships are suggested by Reese & Van Impe (2011).

After the mechanical deformations are obtained via numerical convergence in FEM, the internal forces are used to determine the reaction of the pile on the slipping mass. The location of the reaction is at the intersection of the pile with the slip surface. Based on the lateral analysis, the internal shear forces in each of the lateral directions at the location of the slip surface are applied perpendicular to pile. Similarly, the internal axial force at the slipping location is applied onto the slipping mass towards the longitudinal direction of the pile. Depending on the whether the pile is in tension or compression, the restoring force will be directed into the ground below the slipping surface or in the opposite direction, respectively. Owing to the uncertainty of the actual coupled behavior between the pile, soil and slipping surface in the event of sliding, excluding either of the axial or lateral components can be done by the designer for the purpose of conservative analysis.

4 EFFECTIVE GROUND ANGLE

In cases where the slip surface intersection with the pile is shallow, the lateral soil-pile interaction can be affected by the ground slope. Reese (1958) proposed an effective ground slope parameter which factors the p-y curves based on the geometry of a conical wedge extending above the bottom extent of the soil displacement to the ground surface. It is based on the slope of the ground topography at the head of the pile, as well as the batter angle and direction of the pile, according to Eq. 3 (Reese & Van Impe 2011). Deep failures are not affected by the effective ground slope. *p-y* curves which are independent of effective ground slope can alternatively be specified. The effect of ground slope and batter angle can also be ignored by the practitioner altogether. The original methodology (Reese 1958) applies to 2D analyses of the pile along a vertical section and is extended in 3D for this paper.

$$\theta i = \alpha_i - \beta_i \qquad [3]$$

Where θ_i is the effective ground angle for the lateral pile analysis in the i^{th} lateral axis, and α_i and β_i are the ground angle and batter angle in the direction of the i^{th} lateral axis, respectively.

In the proposed methodology, a vertical plane is taken in each of the principal axis directions of the pile. Solely for the purpose of obtaining the effective ground slope, β_i in each lateral direction is based on the batter angle of the pile projected onto the corresponding vertical plane, and αi is the ground slope along the vertical plane. The determination of g_i and b_i is illustrated in Figure 1, which shows the pile starting from ground surface *G* intersecting the slip surface *S*.

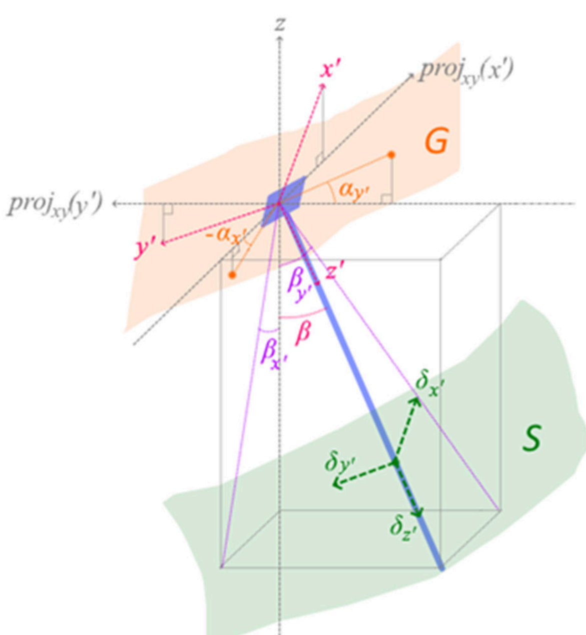


Figure 1. Determination of effective ground angle in the two principal lateral directions

The principal lateral axes of the pile are shown as *x'* and *y'*, while the axial direction of the pile is *z'*. The projection of x' onto the horizontal plane *xy* is denoted $proj_{xy}(x')$, and the projection of *y'* onto the horizontal plane is denoted $proj_{xy}(y')$. A vertical section is taken along *x'*, for which αx is taken as the angle between the horizontal $proj_{xy}(x')$ and the ground surface and β_x is the batter angle if the pile were projected onto this vertical section. The same is done to determine α_y and β_y in the *y'* direction. Note that the ground slope and batter angle are taken positive clockwise from the positive axis direction. β is the true batter angle of the pile, whose magnitude is equal or greater than the projected values β_x and β_y. Finally, the soil displacement δ is partitioned into the three principal components and applied uniformly from the top of the pile to the intersection of the pile with *S*.

5 NUMERICAL EXAMPLE

For the example part of this study, a pile-reinforced slope in investigated using the LIPS-R (Li et al. 2020) approach for MMO problems, shown in Figure 2.

Name	Colour	Unit Weight (lbs/ft3)	Failure Criterion	Cohesion (psf)	Phi (°)
Stratum I (Tn)		115	Mohr Coulomb	155	30
Stratum II (Tm)		115	Mohr Coulomb	160	30
Stratum IIIa (Ta)		115	Mohr Coulomb	130	18

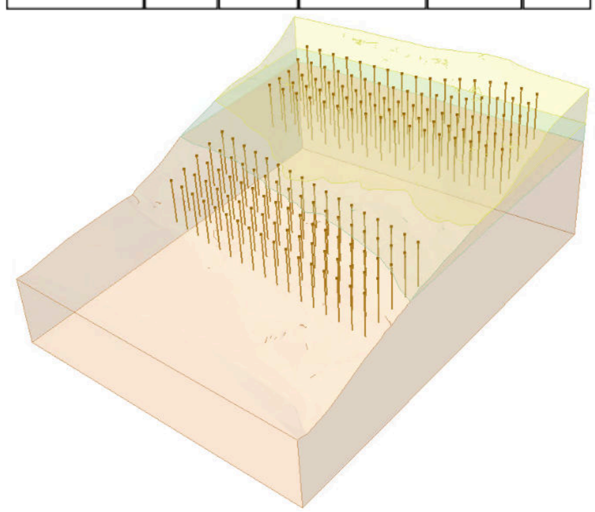


Figure 2. The pile reinforced model and soil parameters used in the study.

Results of the model shown in Figure 2 were computed using the LIPS-R method are shown in Figure 3. Results show two distinct failure modes located at the upper and bottom portions of the slopes with factor of safety of about 1.44. This emphasizes the importance of considering multi-modal optimizations and looking at all the possible failure regions instead of only the most critical one.

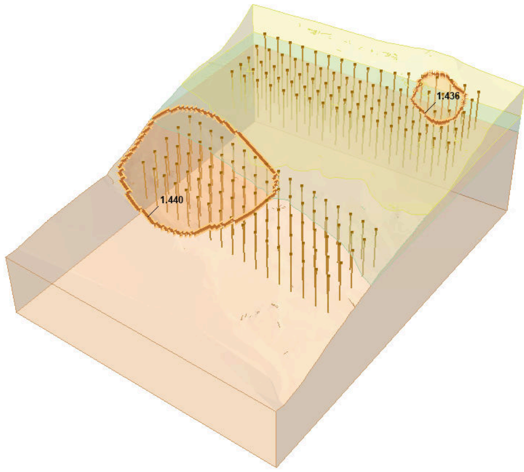


Figure 3. Results of the LIPS-R analysis for the pile-reinforced slope.

The advantage of running the FEM pile analysis is that this method effectively provides a non-linear force diagram be used in the LEM analysis, which is also unique to the direction of slipping, as opposed to a force diagram that is assumed to be constant or independent of the slipping direction.

Figure 4 shows the support resultant forces of the piles which intersect each failure surface. In this figure, the values of the resultant forces are different at different locations of the critical slip surface because of the change of effective ground angle and the depth of the slip surface that intersects the piles.

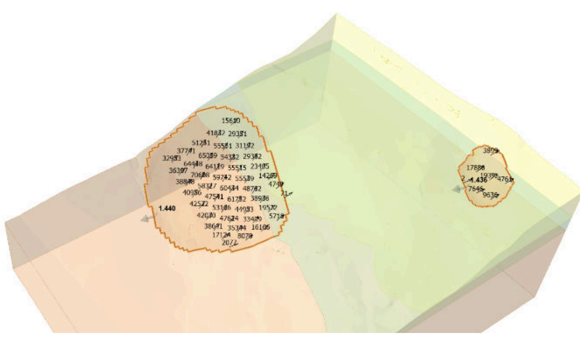


Figure 4. Support reaction forces for both critical slip surfaces.

For the same model presented in Figure 2, three different 2D cross sections are analyzed using MMO analysis. Figure 5 shows this cross sections. These cross sections were selected in a way to pass two MMO regions and include one section through the center of the model.

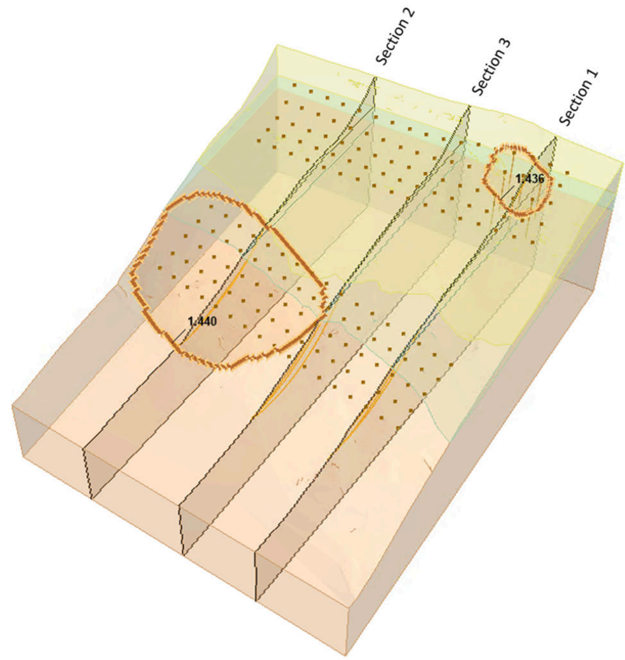
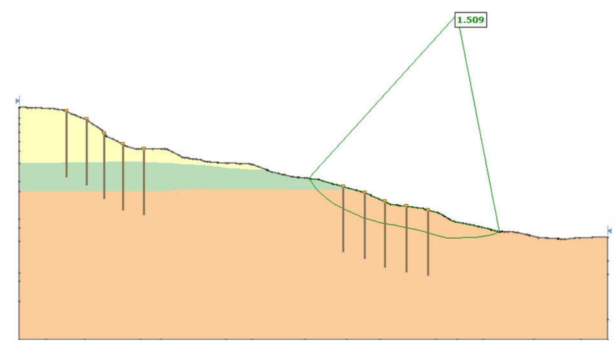
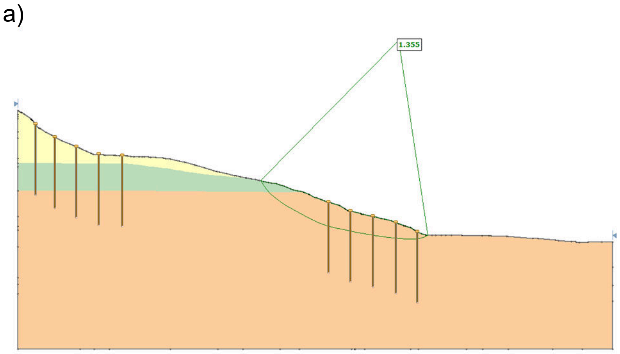


Figure 5. 2D cross sections on top of the 3D model.

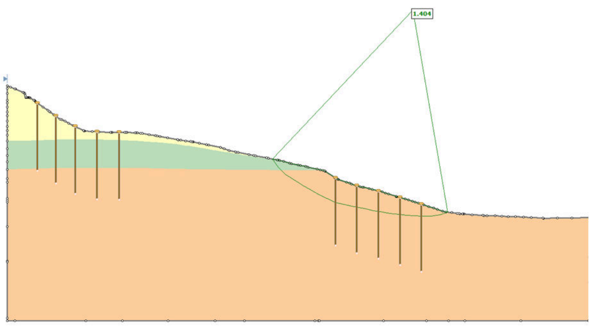
Figure 6a, 6b and 6c shows the results of the 2D analysis run using MMO for sections 1, 2 and 3 respectively.



a)



b)



c)

Figure 6. Results of the 2D MMO analysis for a) section 1 b) section 2 and c) section 3

Factors of safety for Figure 6a, 6b and 6c are 1.51, 1.35 and 1.40, respectively. Figure 6a is passing the center of the 3D north failure and Figure 2 is passing the center of the 3D south failure. 2D FS for sections 1 is slightly higher than the 3D FS of north failure which is 1.44. The reason for that is 2D analysis could not catch the north failure. This might be because of the small size of the north failure region.

On the other hand, in Figure 6b, 2D FS is 1.35 which is lower than the 3D FS of the south failure which is 1.44 and the failure mechanism is the same for both 2D and 3D. Figure 6c is for the center section and it shows 2D Fs of 1.4 which is still lower than the south 3D FS. The results of 2D analysis shows that if the 2D analysis can capture the correct failure mechanism, the FS is lower than 3D. Otherwise, 2D FS might be lower or higher than the 3D FS.

6 CONCLUSION

Multi-modal Optimization (MMO) for the case of a pile-reinforced slope have been used to find the critical slip surfaces and its corresponding factors of safety for 2D and 3D models. The MMO approach provides two different regions for the failure surfaces with the same factors of safety which emphasizes the importance of using MMO approach and the limitations of the traditional LEM approach which only generates one critical factor of safety. 2D MMO results show that 2D analysis may or may not capture the 3D failure region depend on the 2D section created for 2D models.
A coupled FEM pile analysis and 2D and 3D LEM analysis were used to calculate the reaction forces of the piles against the critical slip surface.

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