Comparative assessment of transient creep models to analyze openings in low porosity soft rocks

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ABSTRACT
This article presents a comparative assessment of different types of constitutive model used to represent the time-dependant behavior of soft rocks. The main geomechanical characteristics of these rocks are first summarized, based on experimental observations on the creep of rock salt. Classes of constitutive models are then identified and specific formulations are presented for steady-state and transient creep. Simulations of test results illustrate the main features and differences between different types of constitutive model, including analogical rheologic models, non-linear empirical creep functions, and a unified creep-plasticity formulation with internal state variable (ISV). The discussion also considers the effect of the model on the evolving stress distribution around underground openings. The results and analysis demonstrate some the advantages of using the ISV-SH unified model to represent the hereditary time-dependent behavior of soft rocks.

RÉSUMÉ
Cet article présente une évaluation comparative de différents types de modèles constitutifs utilisés afin de représenter le comportement différé des roches tendres. Les caractéristiques géomécaniques de ces roches sont tout d'abord présentées sommairement à partir d'observations expérimentales sur le fluage du sel gemme. Différentes classes de modèles constitutifs et certaines formulations spécifiques utilisées pour représenter le fluage transitoire et stationnaire sont ensuite présentées. Des résultats d'essais simulés sont utilisés afin d'illustrer les différences entre les divers types de modèle, incluant des modèles rhéologiques analogues, des fonctions de fluage empiriques non-linéaires, et une formulation fluage-plasticité unifiée avec variable interne. On discute aussi l'influence du modèle sur la redistribution des contraintes autour des ouvertures souterraines. Les résultats et l'analyse montrent les avantages associés à l'utilisation du modèle unifié ISV-SH afin de représenter le comportement héréditaire différé des roches tendres.

1 INTRODUCTION

The geomechanical behavior of low porosity soft rocks, such as rock salt and potash, has been extensively investigated over the years. The results from experimental studies conducted in the laboratory under creep (constant stress) conditions and using constant strain rate (CSR) loading have shown that these rocks exhibit a highly non-linear, history- and time-dependent behavior that manifests itself in various ways. For instance, CSR tests on rock salt have demonstrated that it's yield strength (or elastic limit) is very low in a virgin state, with inelastic strains appearing almost from the beginning of deviatoric loading. The inelastic behavior of rock salt is generally ductile (fully plastic, without volume change), as a low confining stress can usually suppress microcracking (see below). Experimental results also show that inelastic strains tend to raise the yield strength and influence the global time-dependent response in a manner that strongly depends on hereditary (memory) effects (Aubertin et al., 1991, 1993, 1999).

A representative constitutive model is required to analyze the time-dependent behavior of these rocks under complex loading conditions, and to obtain adequate stress and strain (or displacement) fields around underground openings. Various models have been developed for this purpose, but questions remain about which one(s) can be deemed appropriate. This article reviews the main categories of constitutive models and present some commonly used equations. The emphasis is placed on the transient inelastic behavior, which constitutes the most challenging aspect to analyze. A comparison between different models is done for typical creep and CSR tests to highlight some of the main differences. The effect of the constitutive model on the progressive stress redistribution around underground openings is also discussed

2 INELASTIC TIME-DEPENDENT BEHAVIOR AND MODELLING

Experimental results obtained on rock salt are used here to describe the behavior of low porosity soft rocks. The specific features of the geomechanical behavior of rock salt have been reported extensively and summarized in various publications (e.g. Senseny et al. 1992; Carter et al. 1993; Aubertin et al. 1999; Aubertin 2020).

Testing of rock salt is typically conducted using triaxial compression tests under constant strain rate (CSR), constant rate of stress (CRS), or constant stress (creep) loading conditions. Relaxation tests can also be used but are seldom conducted in practice. Although CSR and CRS tests conducted at different rates can provide valuable information on the time-dependent inelastic response, creep tests are more commonly used to study this aspect in details (Carter and Hansen 1983; Handin et al. 1986; Cristescu and Hunsche 1998).

Figure 1 shows schematic creep (strain-time) curves for two different stress levels, which can be decomposed into four different straining phases. The pseudo-instantaneous strains during the loading phase are sometimes considered elastic, but typically include also plastic (inelastic) strains when the deviatoric stress exceeds the very low yield strength (< 1 MPa for rock salt in a virgin state). This is followed by up to three explicitly time-dependent phases: transient (or primary) creep, steady-state (or stationary) creep, and accelerating (or tertiary) creep.

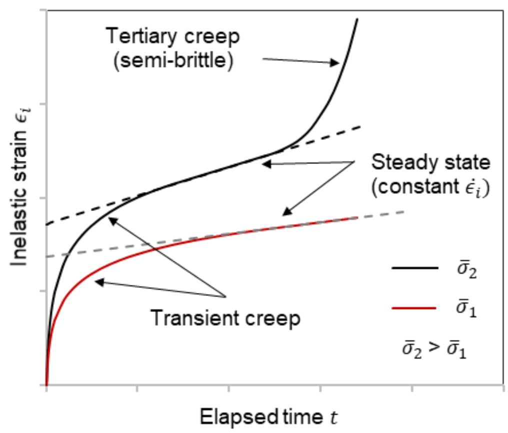

Figure 1: Conceptual creep (strain-time) curves for two deviatoric stresses $\bar{\sigma}$ (MPa). The lower curve (red) corresponds to a stress level $\bar{\sigma}_1$ below the damage initiation (semi-brittle) threshold and only includes transient and steady state creep. The top curve (black), for $\bar{\sigma}_2$, exhibits an additional tertiary creep phase associated with micro-crack (damage) initiation and propagation.

The different strain components can be represented by various types of constitutive models; many of these time-dependent models have been summarized and analyzed in monographs, review papers and books (e.g. Senseny and Hansen, 1987; Aubertin et al. 1987a,b, 1993; Munson and Wawersik, 1991; Cristescu and Hunsche 1998; Julien, 1999; Hampel et al. 2013; Aubertin, 2020).

The traditional approach for constitutive modeling of the time-dependent behavior of rocks is based on a partitioned formulation (written here in incremental form) , where the total strain rate (s^{-1}) is given by

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p + \dot{\varepsilon}^c \qquad [1]$$

with

$$\dot{\varepsilon}^c = \dot{\varepsilon}_t + \dot{\varepsilon}_s + \dot{\varepsilon}_a \qquad [2]$$

where $\dot{\varepsilon}^e, \dot{\varepsilon}^p$, and $\dot{\varepsilon}^c$ are elastic, plastic, and creep strain rates respectively; $\dot{\varepsilon}_t$, $\dot{\varepsilon}_s$, and $\dot{\varepsilon}_a$ are the transient, steady state, and tertiary creep strain rates.

Each component can be described by a distinct function (or law), including for instance linear elasticity for $\dot{\varepsilon}^e$ and classical plasticity with a normality rule for $\dot{\varepsilon}^p$ (Desai and Siriwardane, 1984; Desai and Varadarajan, 1987). The time-dependent (or creep) functions are often formulated as curve-fitting expressions directly related to specific experimental results, such as time- or strain-hardening functions for $\dot{\varepsilon}_t$ and power law equation for $\dot{\varepsilon}_s$ (Carter and Hansen, 1983; Aubertin et al. 1987a,b; Munson and Wawersik, 1991). The tertiary creep term $\dot{\varepsilon}_a$ is frequently omitted for rock salt, as this phase only occurs when the confining stress is low and the deviatoric stress significantly exceeds the damage initiation threshold. The latter is not be considered here, but will be discussed briefly below.

Four broad classes of models can be identified for the time-dependent ductile behavior (Aubertin et al. 1993):

I. Viscous stress models, such as linear viscoelastic models and classical non-linear creep laws;
II. Hereditary integrals models, such as the endochronic theory of creep and viscoplasticity;
III. Mixture theories that scale-up microstructural processes to model macroscopic behavior;
IV. State variable evolutionary models, in which the instantaneous value of the internal and external state variables serve to define the material response as a function of the loading path and mechanical history.

Hereditary (memory) integral models (Class II), such as endochronic models based on an intrinsic time (e.g. Borchert et al. 1984), usually necessitate the knowledge and proper treatment of the entire mechanical history of the material, which is not convenient for large scale problems with complex loading conditions such as underground openings.

Mixture theory models (Class III), often based on micro-mechanisms within the crystalline structure (e.g. Nouailhas et al. 1995; Cailletaud et al. 2003; Pouya et al. 2014), can be very useful to analyze small scale behavior but they show significant limitations for engineering applications, as the upscaling to large volume required to solve geomechanical problems leads to extensive data and computation challenges.

Only the first and fourth classes (I, IV) of models mentioned above are commonly used for rock salt and other low porosity soft rocks. These two classes are considered in more details in the following.

2.1 Viscous stress models

2.1.1 Rheological models

Rheological models based on mechanical analogies were often used in the past (e.g. Serata and McNamara, 1980; Serata 1984; Langer 1984), and these are still sometimes applied to represent the time-dependent behavior of rock salt (and other rocks). However, it is commonly acknowledged that this type of constitutive model doesn't adequately represent the time-dependent response under complex loading conditions, such as for the engineering analysis and design of underground openings. This type of model can be helpful to illustrate the idealized time-dependent behavior in a simplified and intuitive manner, but their basic components (spring, dashpot, and sliding block) have no relevance for the physical processes involved (Lemaitre and Chaboche, 1990). In practice, these models generally require modifications to the basic components equations and the combination of many units (Kelvin or Maxwell) to define so-called extended Burger (or

Bingham) models, hence leading to many (abstract) parameters that are generally difficult to determine experimentally. In addition, these models are not commonly expressed with ordinary differential equations, as is required for general loading conditions, so their use in numerical simulations to analyse the behaviour of openings creates additional challenges. Some of the specific limitations of this type of model are illustrated below.

2.1.2 Partitioned creep models

Using the traditional partitioning approach, depicted by Equations (1) and (2), implies that the inelastic strain can be divided explicitly into two distinct and independent components, namely instantaneous plastic strains and delayed creep strains. However, theoretical and experimental evidences indicate that interaction exists between these two inelastic strain components, such as an effect of creep strain on the yield strength and the influence of the loading rate on the stress-strain curve and on the subsequent strain rate in a creep test (Aubertin et al., 1991, 1993, 1999). These observations are consistent with those made on other ductile crystalline materials, where plasticity and creep strains have been shown to interact (Ohashi et al., 1986).

As indicated above, the elastic behavior in Eq. 1 is typically expressed using Hooke's law (linear, isotropic), which is sometimes applied (incorrectly) to the (pseudo-instantaneous) loading stage in a creep test. The plastic component in Eq. 1 is generally neglected when the focus is on time-dependent behavior. Otherwise, the plastic and creep components should be coupled as both inelastic strain components interact. Omitting $\dot{\varepsilon}^{p}$ thus simplifies the analysis, which would otherwise require the establishment of a relationship between plastic and creep strain components.

Alternatively, unified creep-plasticity formulations can also be used, as will be shown below.

As was also mentioned previously, the creep phases in Eq. 2 can be described by distinct functions (or laws), combined as independent components for transient and steady-state creep. In many instances, the two creep terms are expressed from relatively simple empirical equations based on external variables, i.e. stress, time, and strain (e.g. Boyle and Spence, 1983; Skrzypek and Hertnarski, 1993; Aubertin et al. 1993). A few of these models are available in commercial codes, and they are often used for practical engineering applications. However, these have serious limitations when dealing with complex loading conditions, as will be shown below.

Steady-state creep laws

The analysis of underground excavations in rock salt is often focusing on the steady-state flow, associated with long-term behavior, which is usually considered to be independent of the stress path and strain history (in the ductile regime).

The most often used constitutive model for steady-state creep is the classical Norton power law, which can be written as (Carter and Hansen, 1983):

$$(\dot{\varepsilon}_e)_s = B\sigma_e^n \qquad [3]$$

Where $(\dot{\varepsilon_e})_s$ is the steady-state creep rate (s^{-1}), expressed using the equivalent von Mises strain; σ_e is the von Mises deviatoric stress (MPa*);* *B* and *n* are material parameters. This model is often used in salt mining applications because of its simplicity, although it bears significant limitations as discussed below (Aubertin and Aubertin, 2020).

Exponent *n* in this equation is sometimes considered a material constant, but it's actual value tends to increase with the magnitude of the deviatoric stress (e.g. Cristescu and Hunsche 1998; Aubertin et al. 1999). The specific power law equations presented in the next section exemplifies this aspect. Alternatively, the power law can be replaced by an exponential function of the deviatoric stress (Aubertin et al. 1999) or by the hyperbolic sine law introduced below (Borm and Haupt, 1988; Aubertin et al. 1998; Julien, 1999).

Empirical transient creep models

Transient creep laws are generally expressed in terms of time (t) or equivalent (isotropic) strain (ε_e), leading to classical time- or strain-hardening functions (Boyle and Spence, 1983; Skrzypek and Hertnarski, 1993). The physical processes involved and experimental evidence show however that time is not an appropriate variable to represent the complex transient response. Cumulative inelastic strain better represents hardening behavior during simplified incremental creep tests, but it is usually insufficient to account for a complex mechanical history. The empirical non-linear formulations commonly used for transient creep have been shown to be inadequate for rock salt, and similar rocks (and other crystalline materials), when the stress path and loading history is elaborate (Mukarami et al. 1986; Handin et al., 1986; Munson and Wawersik, 1991; Aubertin et al. 1991; 1993. 1999).

In addition, the separation between the transient and steady-state creep phases may be mathematically convenient, but it is physically unsound because the same physical processes govern both of these time-dependent inelastic phases.

Results shown below illustrate some of the limitations of this type of model formulation.

2.2 Internal state variable models

A powerful approach to develop inelastic constitutive models is based on adding one or more internal state variables (ISV) in the differential equations to represent rate and loading history dependencies (Lemaitre and Chaboche, 1990; Aubertin et al. 1991, 1993, 1999; Cristescu and Hunsche 1998). With ISV models, the value of each internal state variable is defined by an evolution law (differential equation) that typically contains hardening and recovery terms (following the Bailey-Orowan principle).

The evolution law thus specifies how the ISV changes as a function of the loading path and history. Each ISV can be related to active (micro) mechanisms that control inelastic straining, so the model formulation is usually closer to the physical inelastic flow processes; this is an advantage when analyzing problems that involve loading conditions that differ from the available experimental data.

In addition to evolution laws, an ISV model includes a kinetic law that represents the material time-dependent response at a constant structure or state (Aubertin et al., 1991). When combined with evolutionary ISV, the kinetic law can describe the entire spectrum of inelastic strains, including plastic, transient and steady-state creep,

The ISV constitutive modelling approach has been commonly used for metallic materials (Miller 1987; Lemaitre and Chaboche 1990; Krausz and Krausz 1996; Besson et al. 2001; Lubliner 2008). Some of the main ISV models developed for rock salt have been reviewed by Aubertin et al. (1993, 1998), Cristescu and Hunsche (1998), Julien (1999), Hampel et al. (2013), Reedlun (2018), and Aubertin (2020).

2.2.1 The SUVIC model

The SUVIC model was developed specifically for the inelastic behavior of rock salt (Aubertin et al. 1991, 1999; Julien, 1999; Yahya et al. 2000); SUVIC stands for "Strain rate history dependent Unified Viscoplastic model with Internal state variables for Crystalline materials". The formulation of this unified model, which has evolved over the years, uses up to four ISV to describe mixed (kinematic and isotropic) hardening; a damage variable can also be included (Aubertin et al. 1998, 1999; Julien 1999), but this component is not considered here. SUVIC is based on the unified theory framework for plasticity and creep, which implies that the mechanical state is entirely described by the current values of the external (observable) state variables (i.e. stresses, strains and temperature) and of the selected ISV.

The SUVIC model has been shown to provide a good representation for the geomechanical response of rock salt under different loading conditions including plasticity, creep and relaxation (Aubertin et al. 1999; Julien, 1999; Yahya et al. 2000), and it has been recognized as a significant development and influential contribution in the field (e.g. Cristescu and Hunsche 1998; Besson et al. 2001; Reedlunn, 2022).

This constitutive model considers that total strains have an elastic and a viscoplastic (inelastic) components. For small perturbations under isothermal loading in the ductile regime, the total strain rate tensor is expressed as the sum of elastic and inelastic strain rate tensors:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{e}_{ij} + \dot{\varepsilon}^{i}_{ij} \quad [4]$$

The elastic strain rate tensor is given by the generalized Hooke's law for isotropic materials.

Under uniaxial stress, the basic equation used for the inelastic flow law is given by (Aubertin et al. 1991):

$$\dot{\varepsilon}^{i} = A\left\langle\frac{\sigma-\sigma_i}{K}\right\rangle^{N} \quad [5]$$

with $\sigma_i = B + R$ [6]

where $\dot{\varepsilon}^{i}$ is the inelastic strain rate, A and N are material parameters, B, R and K are internal state variables, and σ_i is the internal stress. The model can also be written in a tensorial form (Yahya et al. 2000).

Each ISV evolves through competitive processes that can be expressed as follow, for a generic internal state variable $Y \in (B, R, K)$.

$$\dot{Y} = f_h - f_d - f_s \quad [7]$$

where f_h, f_d, and f_s are respectively the terms that account for strengthening (hardening) mechanisms and for the dynamic and static recovery associated with softening processes. The hardening f_h and dynamic recovery f_d terms both evolve with the inelastic strain, while the static recovery f_s term evolves with time and cumulated value of *Y*. All these terms are also temperature dependent, but this aspect is not considered here. The specific functional forms for each ISV are given by Aubertin et al. (1999) and Yahya et al. (2000).

Within the adopted modeling framework, steady-state flow that may follow transient behavior is independent of previous mechanical events, or memory effects. Therefore, it can be formulated using external variables only. For rock salt, the most representative relationship between the von Mises equivalent inelastic strain rate and stress is given by the hyperbolic sine function given below.

The SUVIC model is applied here to creep tests results on rock salt.

2.2.2 The ISV- SH model

Despite its advantageous capabilities, the use of the SUVIC model raises non-trivial challenges due to the relatively large number of variables and parameters that must be evaluated, which sometimes require non-conventional testing (Aubertin et al. 1999). The numerical treatment of the highly non-linear and coupled differential equations that must be solved to conduct simulations can also create significant challenges (Julien et al. 1998; Yahya et al. 2000; Aubertin et Julien, 2015).

A simplified version of this model was thus considered for practical engineering applications, particularly for the analysis of underground mine openings (Bouliane et al. 2004). The corresponding ISV-SH model was recently implemented in FLAC (Itasca Consulting Group, 2019) to conduct simulations (Aubertin et al. 2018; Aubertin et Aubertin, 2021). The main equations of this model are formulated as follow:

$$\dot{\varepsilon}^{i} = A\left(\frac{\bar{\sigma}}{K}\right)^{N} \quad [8]$$

$$\dot{K} = A_5\left(1 - \frac{K}{K'}\right)\dot{\epsilon}^{i} \quad [9]$$

$$K' = \frac{\bar{\sigma}'}{\left(\frac{\dot{\varepsilon}^{i}}{A}\right)^{\frac{1}{N}}} \geq 1\ MPa \quad [10]$$

$$\bar{\sigma}' = \sigma_0 \cdot \sinh^{-1}\left(\frac{\dot{\varepsilon}^{i}}{\dot{\epsilon}_0}\right)^{\frac{1}{n}} \quad [11]$$

where $A, A_5, N, n, \sigma_0, \text{and } \dot{\epsilon}_0$ are the 6 material parameters determined from experimental results; $\bar{\sigma}'$ and K' are the saturated (steady-state) values of the deviatoric stress $\bar{\sigma}$ and the isotropic internal state variable K. The evolution of K (≥ 1 MPa) progressively converges towards K' as the material approaches steady state flow.

The ISV-SH model initially behaves as a classical strain-hardening law at small inelastic strain in a creep test. But contrary to the latter, it naturally evolves towards a stationary creep condition at larger strain.

Figure 2 shows experimental creep tests results on rock salt at different stress levels (with the loading stages under CRS conditions), and simulated curves comparing the ISV-SH and the SUVIC models responses. As can be seen, the SUVIC model follows the testing results very closely (Yahya et al. 2000). The ISV-SH model also approximates the creep behavior quite well, including the loading steps, but the match is less precise than with the SUVIC model. This can be expected as the ISV-SH model contains fewer variables (a single ISV instead of 4) and a much smaller number of adjustable parameters (6 vs 17) than SUVIC.

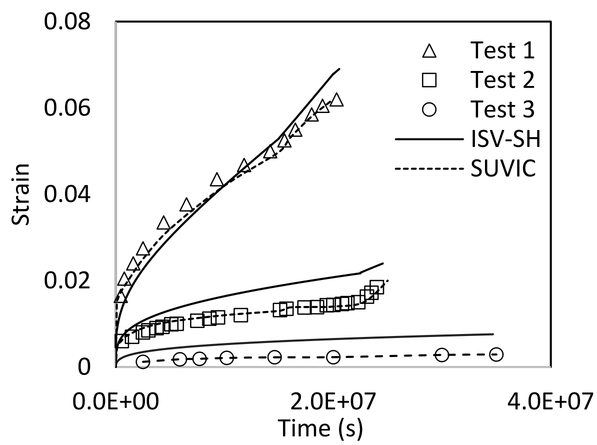

Figure 2: Constant stress (creep) test strain-time curves obtained experimentally and from simulations with the ISV-SH and SUVIC models; the data points were obtained from tests on Avery Island rock salt (details on experimental and SUVIC results are provided by Yahya et al. 2000). Stress levels for Test 1: 15, 16 MPa ; Test 2: 10, 12.5 MPa Test 3: 5 MPa.

3 APPLICATION AND COMPARISON

Specific Class I and IV model formulations are presented and applied below to illustrate some of their main features. The comparison made for specific loading conditions also highlights key differences.

3.1 Visco-elastic rheological models

The Burger model represents the combination in series of a Kelvin and a Maxwell assembly. The differential equation partitioning of the Burger model can be expressed as (Lemaitre and Chaboche, 1990):

$$\dot{e_{ij}} = \dot{e}_{ij}^K + \dot{e}_{ij}^M \qquad [12]$$

where $\dot{e}_{ij}^K$ and $\dot{e}_{ij}^M$ are the Kelvin and Maxwell strain rate components. These deviatoric strain rates are given by the following equations:

$$\dot{e}_{ij}^K = (S_{ij} - 2G^K e_{ij}^K)/2\eta^K \qquad [13]$$

$$\dot{e}_{ij}^M = \frac{\dot{S_{ij}}}{2G^M} + \frac{S_{ij}}{2\eta^M} \qquad [14]$$

where S_{ij} is the deviatoric stress tensor, $\dot{S_{ij}}$ is the deviatoric stress rate tensor; $\dot{e}_{ij}^K$, e_{ij}^K, and $\dot{e}_{ij}^M$ are deviatoric tensor for the strain and strain rates of the Kelvin (K) and Maxwell (M) components; G and η are the corresponding stiffness moduli and viscosity coefficients.

Eq. 12-14 lead to the following steady state strain rate $\dot{e}_{ij}^{SS}$, controlled by the Maxwell component:

$$\dot{e}_{ij}^{SS} = \frac{S_{ij}}{2\eta^M} \qquad [15]$$

The Burger model formulation can also be expressed explicitly to represent the strain-stress-time behavior during a constant stress (creep) test as:

$$\varepsilon_a(t) = \frac{2\sigma_a}{9K} + \frac{\sigma_a}{3G^M} + \frac{\sigma_a}{3G^K}\left[1 - \exp\left(-\frac{G^K}{\eta^K}t\right)\right] \qquad [16]$$

The parameters for the Burger model (Eq. 16, Table 1) , calibrated from one of the creep curves in Figure 2 (stress of 15 MPa), are used in subsequent comparative analysis.

3.2 Steady state equations

As indicated above, the exponent of the power law (PL) equation (Eq. 3) tends to vary with the stress state. The Norton power law (PL) formulation considered here includes two components:

$$\dot{\varepsilon}^{ss} = a_i \bar{\sigma}^{n_i} \quad \Big\} \begin{matrix} \bar{\sigma} \le \sigma_{ref}, i = 1 \\ \bar{\sigma} > \sigma_{ref}, i = 2 \end{matrix} \qquad [16]$$

This leads to a bilinear representation in the log – log plane, as shown in Figrue 3 (with exponents n_i). The parameters given in Table 2 Table 2: Parameters for the one and two components power law (PL) and hyperbolic sine function, calibrated from steady-state creep tests results on Avery Island (AI) rock salt. were used to draw the lines for a single PL and for the two components of Eq.16.

A more appropriate steady state equation, which better reflects the actual strain rate dependence on the deviatoric stress, can be formulated with the following hyperbolic sine law (Aubertin et al. 1998; Aubertin and Julien 1995; Aubertin and Aubertin, 2021):

$$\dot{\varepsilon}^{ss} = \dot{\varepsilon_0} \cdot \sinh^n\left(\frac{\bar{\sigma}'}{\sigma_0}\right) \qquad [17]$$

This steady-state creep function is also shown in Figure 3, based on the parameters given in Table 2. This figure also includes experimental results obtained on Avery Island (AI) rock salt (taken from Carter et al. 1993; Senseny et al., 1993; Julien 1999).

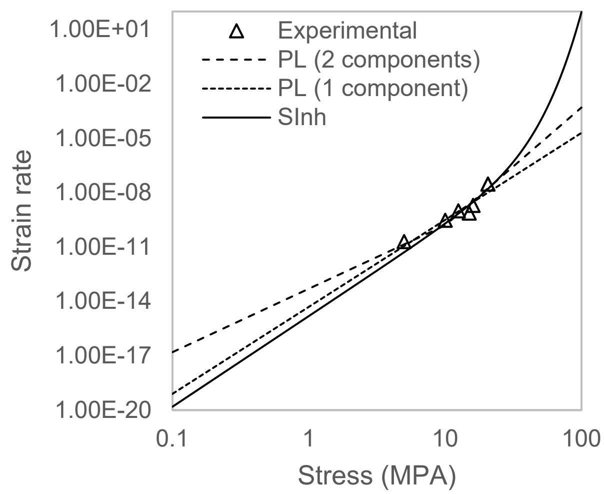

Figure 3: Steady state strain rate (s^{-1}) for the single and two components power law (PL) models and hyperbolic sine (sinh) model, calibrated for Avery Island (AI) rock salt (experimental data also shown).

3.3 Transient creep models

Conventional time and strain hardening are among the most often used transient creep formulations for ductile materials..

The time hardening equation can be expressed as (Boyle and Spence, 1983):

$$\dot{\varepsilon} = mBt^{m-1}\sigma^{n} \quad [18]$$

where m, B and n are material parameters, and t is time.

Eq. 18 can be integrated and formulated as a function of the cumulative strain. The corresponding strain hardening equation then becomes :

$$\dot{\varepsilon} = mB^{\frac{1}{m}}\sigma^{\frac{n}{m}}/\varepsilon_{i}^{\frac{1-m}{m}} \quad [19]$$

The material parameters, given in Table 3, are the same for Equations 18 and 19, but the two models behave quite differently, as shown below; these parameters were calibrated for the transient component of the AI creep test at 10 MPa.

3.4 The ISV-SH model

This ISV-SH model presented above (Eq. 8-11) includes a single ISV to represent isotropic hardening in the kinetic law. It can represent transient and steady-state creep (with the loading phases) as shown in Figure 2. Table 4 gives the parameters for this model obtained from experimental results on Avery Island rock salt shown in this figure. The same parameter values are used for the ISV-SH model and the sinh steady-state creep model (Eq. 17) .

Table 1: Burger model parameters (Eq. 13-16) for AI rock salt, calibrated for the 15 MPa creep test (Figure 2).

Parameter	Value
G^K	600 MPa
η^K	3 GPa · Day
G^M	17 GPa
η^M	8 GPa · Day

Table 2: Parameters for the one and two components power law (PL) and hyperbolic sine function, calibrated from steady-state creep tests results on Avery Island (AI) rock salt.

Parameter	Value
Power Law (2 components)	
a_1	5E-14 s^{-1}
a_2	5E-17 s^{-1}
n_1	3.5
n_2	6.5
σ_{ref}	10 MPa
Power Law (1 component)	
a	5E-15 s^{-1}
n	4.8
Hyperbolic Sine Function	
$\dot{\epsilon}_0$	3.0E-09 s^{-1}
σ_0	18.1 MPa
n	5.0

Table 3: Parameters of the Time-hardening (TH) and Strain-hardening (SH) models (Eq. 18, 19), calibrated on the creep test on AI rock salt at 10 MPa.

Parameter	Value
B	1.50E-8
m	0.5
n	2.5

Table 4: ISV-SH model parameters for the creep of AI salt.

Parameter	Value
$\dot{\epsilon}_0$ (s-1)	3.0E-09 s^{-1}
σ_0 (Pa)	1.81E+07 Pa
n	5.0
A (s-1)	5.0E-12 s^{-1}
A_5	5.0E+08
N	5
K_0 (Pa)	1.0E+06 Pa
E (GPa)	17.06 GPa
ν	0.33

4 SIMULATED RESPONSES FOR SPECIFIC LABORATORY TESTS

Differences between the constitutive models presented above are illustrated through simulations of experimental tests. Two types of laboratory experiments were simulated by implementing the constitutive models as differential equations in a Python compiler. The simulated tests are:

i. Single stage and incremental creep tests at two deviatoric stress levels of 10 and 15 MPa (loading rate of 10 kPa/s).
ii. Constant strain rate (CSR) tests with two imposed strain rates of 1.0E-7 and 1.0E-5 s^{-1} .

The comparison shown here involves the Time hardening (Eq. 18), Strain hardening (Eq. 19), Burger (Eq. 12-16), and ISV-SH (Eq. 8-11) models, implemented to conduct the two simulations. Figures 4 and 5 present the simulated curves for the creep and CSR tests, respectively.

The simulated single stage and incremental creep tests presented in Figure 4 show almost similar curves for transient creep with the 4 models at a stress of 10 MPa ; this is expected because this creep curve obtained with ISV-SH has been used to calibrate the parameters of the different models. The TH and SH incremental creep curves then diverge noticeably during the second loading stage (instantaneous with these models), as is typically observed for these two transient creep models (Boyle and Spence, 1983; Aubertin et al. 1991, 1999). The curves given by the ISV-SH and SH models are close to each other, which is also expected because both tend to follow typical incremental (transient) creep test results. The TH and Burger models however differ noticeably, and are thus deemed unable to represent the rock salt transient creep behavior.

The CSR test simulations shown in Figure 5 illustrate the influence of the imposed strain rate for the four constitutive model formulations. The ISV-SH response shows a non-linear stress-strain curve which tends to converge towards a constant stress as the ISV reaches saturation (at K'). These curves follow the experimental trends for CSR tests observed on rock salt (e.g. Handin et al. 1986; Aubertin et al. 1991, 1999). It is also seen that the TH, SH and Burger models cannot appropriately represent the influence of strain rate due to their restrictive (creep based) formulations, which don't lead to an appropriate steady-state flow at the corresponding saturation stress. These 3 constitutive models thus fail to capture the non-linear stress-strain and strain rate relationship of rock salt.

These comparative results illustrate some of the main characteristics of the 4 constitutive models considered here, and highlight limitations of commonly used transient creep models.

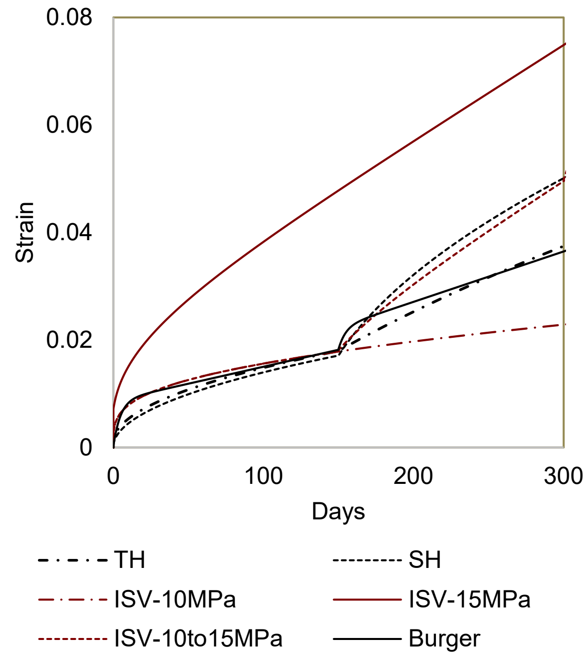

Figure 4: Single stage and incremental creep tests simulated with the Time Hardening (TH), Strain Hardening (SH), Burger, and ISV-SH models; deviatoric stresses of 10 MPa and 15 MPa.

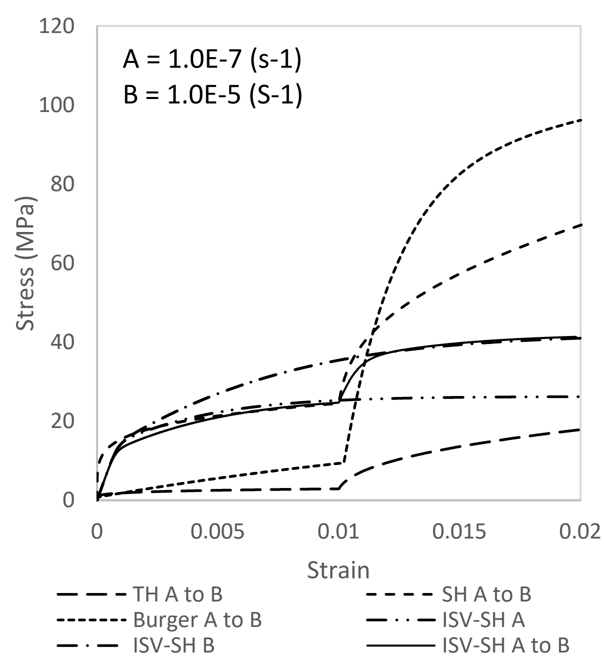

Figure 5: Constant strain rate (CSR) test simulated with the Time Hardening (TH), Strain Hardening (SH), Burger, and ISV-SH models. Constant strain rate of 1.0E-7 and 1.0E-5 s^{-1}) .

5 EVOLVING STRESS DISTRIBUTION AROUND OPENINGS

The analysis, design, and monitoring of underground excavations in creeping rocks, such as rock salt and potash, involve the evolving stress and displacement fields. Numerical simulations conducted for this purpose produce results that are directly related to the applied constitutive model.

The selection of a constitutive model should target a mathematical formulation that is as simple as possible, but which must adequately represent the material behavior for the relevant loading conditions. For engineering problems involving extended time scales and complex loading conditions, a thorough understanding of the material inelastic behavior becomes a prerequisite to select an appropriate constitutive model.

The use of a linear viscous stress, or viscoelastic (Class I) rheologic model to analyze an unsupported underground opening leads to a constant stress field that develops instantaneously (when there is no viscoplastic threshold). The viscoelastic flow of the material around the excavation then proceeds without any modification to the stress field, either in time or in space. As was shown here in Figures 5 and 6, and in many others publications, this type of model cannot describe the actual behavior of rock salt even under relatively simple loading conditions. This type of model is inappropriate for even simple applications in rock salt (Fuenkajom and Daemen, 1988; Aubertin et al. 1993).

Time-dependent models that consider non-linear stress-strain relationships for the rock mass generate a stress redistribution around underground openings, even for a constant far field stress state. Such a stress redistribution is required to ensure the continuity of the strain field. As the stress redistribution around an unsupported opening occurs, part of the higher deviatoric stresses initially imposed near the excavation walls is gradually transferred away, toward the regions that are initially less loaded. The stress state then tends progressively toward a stationary condition. This stress redistribution induces a transient inelastic behavior in the structure, with the wall convergence rate decreasing progressively with time. Because the general shape of the convergence curve shows a transient phase resembling that of laboratory creep tests, it has sometimes given rise to ambiguities regarding the distinction between the transient inelastic flow of the material, which is related to its hardening (as is the case in the primary creep phase in a laboratory creep test) and the transient behavior of the structure that also depends on the stress redistribution. During this redistribution period, inelastic straining is related to both the varying stress state and the inherent transient creep behavior. The former depends on the excavation geometry and stress-strain (and strain-rate) non-linearity, while the latter stems from hardening of the material itself. Both aspects are related and interact in a complex manner, as each change of stress produces transient inelastic flow of the material, leading to further stress redistribution because the hardening of the material influences its stress-strain (and stress-strain rate) relationship.

Numerical simulation must consider the interaction between these two complementary processes, which both influence the rate at which the stresses are redistributed around the excavation. Recent simulation results obtained by the authors demonstrate that a full analysis with the ISV-SH model generally produces a longer period of noticeable stress variation in the structure when compared to what is obtained by using a classical creep law (see also Morgan and Krieg, 1988; Aubertin et al., 1993).

An elastic/steady-state creep law, such as the Norton power law function, produces a transient inelastic flow of the structure that is related to the stress redistribution only. Simulations with such model show that there is a progressive migration of the peak deviatoric stress farther from the excavation walls as the inelastic flow proceeds (e.g. Aubertin and Aubertin, 2021). But such relatively simple model cannot give satisfactory results when considering commonly accepted predictive technology assumptions (e.g. Munson and Wawersik, 1991). When compared to the actual behavior of underground openings, the difference between the simulated and observed wall displacements is then often very large (> 100-200%; Morgan and Wawersik, 1989; Bérest, 1991). This is not surprising because such type of model completely neglects the transient (hardening) inelastic behavior of the material, and its interaction with the stress redistribution.

It is also interesting to recall here that a stationary creep rate around underground openings has seldom been formally identified, even for very old workings (Borm and Haupt, 1988), indicating that a steady-state stress field may take a very long time to develop in the surrounding rock. In standard laboratory tests however, isostatic loading conditions usually lead to steady-state flow within a few days to a few months for rock salt, depending on the imposed temperature, stress, and/or strain rate.

The use of a steady-state creep law is thus not representative of the reality. It can however be applied to develop analytical solutions for idealized geometric structures and for stationary stress distribution (Boyle and Spence, 1983), which can then be used to investigate numerical modeling results and to consider the general effects of specific factors on the overall behavior of underground openings.

Despite the caveats and limitations mentioned above, it may still be tempting to use empirical elastic/secondary creep law to describe the observed wall convergence of specific underground excavations, under a given set of geometrical and boundary conditions, by adjusting the model parameters (such as the elastic modulus E and the power law exponent n) to fit empirically in-situ rather than experimentally controlled laboratory data. This approach doesn't however follow a predictive technology procedure for excavations in rock salt, where in-situ measurements can serve only to validate the model and calculations, but not to correct the material constants (Munson and Wawersik, 1991). Ignoring the importance and characteristics of the history depend transient behavior in the analysis and design of underground openings or when monitoring existing underground excavations can lead to important misinterpretations.

The addition of an empirical transient creep law such as the time or strain-hardening functions presented above, used by many authors in partitioned non-linear viscous stress (class I) models, lead to limited improvement of the simulation results for underground openings. The external variables used in these models, i.e. time or equivalent (isotropic) strain, does not allow a proper consideration of the evolutionary state of the material that occurs during transient inelastic flow, and the ensuing effect on the stress-strain (and stress-strain rate) relationship. Thus, the complex interaction between the stress redistribution phenomenon and hardening of the material is not represented appropriately. The inadequacy of the constitutive models means that a large portion of the discrepancy between predicted and instrumented behavior of underground excavations in rock salt (and similar rocks) must be attributed to the constitutive model used, when these are compared to the real (physical and phenomenological) time-dependent inelastic behavior (Côme, 1990).

It is thus important to include in a more rigorous fashion the transient inelastic behavior in the constitutive model, to correctly represent the time- and history-dependent response of underground excavations. As the stress path and the strain (and strain rate) history are quite different for laboratory testing and in-situ loading conditions, the use of constitutive models closely related to the actual deformation mechanisms is required. Internal state variable models such as ISV-SH are deemed more appropriate for engineering needs, as these allow a unified representation of the inelastic flow that encompass steady-state and transient behavior. This model takes into account the fact that any modification to the loading conditions (in the lab or in the field) leads to a transient behavior of the material. Memory effects are then represented by the evolving value of the ISV given by properly formulated differential equations.

6 DISCUSSION AND CONCLUSION

Internal state variable models constitute a powerful tool to assess the stress, strain and displacement fields around underground openings. The ISV-SH model formulation presented here offers advantageous capabilities, but its formulation can also be improved to address more specific responses. For instance, it's not entirely clear if expressing the strain rate as a function of the von Mises deviatoric stress adequately represents the actual material behavior. This isotropic model also neglects the contribution of kinematic hardening, which may become important for conditions that lead to a significant rotation of the principal stresses.

It should also be emphasized again that the application of predictive technology to underground excavations should be considered with great care for model validation. The validation of a constitutive model should first be thoroughly conducted by using well-controlled laboratory test results, starting with isostatic loading conditions, and then considering instrumented hyperstatic structures such as thick walled cylinder experiments (Morgan and Wawersik, 1991; Julien et al. 1998; Aubertin and Julien 2015). Confidence can thus be gained before applying in-situ predictive technology, where the apparent accuracy of the results can be misleading.

Another aspect o consider is related to observations showing that underground excavations in rock salt often induce fracture near the opening walls, where the initial deviatoric stress is high and the mean stress is low. A few existing models consider this type of inelastic behavior of rock salt (e.g. Cristescu and Hunsche, 1998; Aubertin et al., 1998), but the effect of damage on time-dependent behavior should constitute one the focal points for future developments.

There are thus still many challenges associated with the selection of an appropriate constitutive law for the prediction of the stress, strain and displacement fields around underground openings in rock salt (and other low porosity soft rocks).The inelastic behavior of crystalline materials, such as rock salt and potash, is inherently complex and very sensitive to mechanical history, so that empirical models based on curve fitting expressions of experimental results obtained under simple loading conditions are usually inadequate. A valid constitutive model should be based on sound physical processes, so that confidence can be placed on laboratory test results before these are extrapolated to in-situ conditions over long periods of time.

It cannot be expected that a constitutive model will be able to reproduce all the different characteristics of the mechanical behavior of rock salt and similar materials. It should however be able to describe the key features for the loading conditions relevant to the problem being analyzed.

For the design of underground excavations, the interaction between the stress redistribution within the structure and the transient (hardening) inelastic behavior of rock salt strongly suggest an internal state variable model be considered.

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