VOID RATIO CONSTITUTIVE RELATIONSHIP FOR EXPANSIVE SOILS BASED ON SINGLE STRESS STATE VARIABLE FRAMEWORK

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ABSTRACT
The generalized consolidation theory based on two-stress state variable framework has been utilized in many previous studies to predict the volume change behavior of unsaturated soils under mechanical stress and matric suction change. This approach requires a constitutive surface that relates the soil void ratio to net stress and soil matric suction. Several mathematical functions have been proposed to describe the void ratio constitutive surface. The constants of void ratio surface function for a given soil are normally determined by fitting the equation to the laboratory test data. High number of fitting parameters is typically used to obtain a two-variable void ratio surface function with sufficient degree of approximation. In this study, a single stress state variable framework was adopted to describe the void ratio as a function of effective stress for soils in unsaturated state. The proposed approach was used to fit single variable void ratio curves to the laboratory test data for two different expansive soils under a range of mechanical stress and matric suction using fewer fitting parameters.

RÉSUMÉ
La théorie de la consolidation généralisée basée sur un cadre de variables d'état à deux contraintes a été utilisée dans de nombreuses études antérieures pour prédire le comportement de changement de volume des sols insaturés sous contrainte mécanique et changement d'aspiration matricielle. Cette approche nécessite une surface constitutive qui relie le rapport de vide du sol au stress net et à l'aspiration matricielle du sol. Plusieurs fonctions mathématiques ont été proposées pour décrire la surface constitutive du rapport de vide. Les constantes de la fonction de surface du rapport de vide pour un sol donné sont normalement déterminées en ajustant l'équation aux données d'essai en laboratoire. Un nombre élevé de paramètres d'ajustement est généralement utilisé pour obtenir une fonction de surface à rapport de vide à deux variables avec un degré d'approximation suffisant. Dans cette étude, un cadre unique de variables d'état de contrainte a été adopté pour décrire le rapport de vide en fonction du stress efficace pour les sols à l'état insaturé. L'approche proposée a été utilisée pour adapter les courbes de rapport de vide variable unique aux données d'essai en laboratoire pour deux sols expansifs différents soumis à une gamme de contraintes mécaniques et d'aspiration matricielle en utilisant moins de paramètres d'ajustement.

1 INTRODUCTION

Volume change behavior of expansive soils is not only a function of mechanical loading, but is also dependent on changes in soil moisture content. The soil swelling/shrinkage behavior is a major concern for geotechnical engineers in particular where lightly loaded structures and underground utilities are supported by expansive clays. Therefore, accurate prediction of expansive soil deformation response to soil moisture changes is an important task in the geotechnical engineering analysis.

In the past decades, several models have been developed to simulate the deformation behavior of unsaturated expansive soils. Most of these models are within the framework of two stress state variables unsaturated soil mechanics (e.g. Briaud et al. 2003, Vu and Fredlund 2004 and 2006, Adem and Vanapalli 2013a, Ito and Azem 2014, Karunarathne et al. 2018). In instances, when the expansive soils undergo both mechanical and environmental loadings, these models require constitutive relationships that relate the deformation state variable (e.g. void ratio, e) to two stress state variables i.e., net normal stress and matric suction. The constitutive relationship is normally obtained by fitting a surface (two-variable function) to the laboratory testing data from controlled net total stress and controlled soil suction conditions. The best fitted constitutive surfaces are commonly obtained using mathematical functions with several empirical parameters (e.g. Vu 2003, Zhang 2004).

In addition to multiplicity of the fitting parameters used in constitutive surface equations, the two-state stress framework cannot be utilized within the context of classical soil mechanics that considers effective stress as the single stress state variable. Therefore, many researchers have been investigating the extended forms of the conventional single-stress state (effective stress-based) framework for unsaturated soil conditions (e.g., Khalili and Khabbaz 1998; Lu and Griffiths 2004; Lu and Likos 2006; Alsherif and McCartney 2014; Manahiloh et al. 2016).

In this study, a single stress state variable framework originally proposed by Lu and Likos (2006) was adopted to describe the void ratio as a function of effective stress for soils in unsaturated state. The proposed approach was used to fit a single variable void ratio curve to the laboratory test data for two different expansive soils for a range of mechanical stress and matric suction.

The proposed constitutive relationship (void ratio vs effective stress) was used to model a benchmark problem for expansive soil Regina clay subjected to a constant infiltration rate for 175 days (Vu 2003). The results were compared with those obtained by some of the previous studies. For numerical simulation of this problem, a two-dimensional variably saturated flow finite element model was employed to model the moisture content distribution within the soil mass. The results were then used in a one-dimensional deformation analysis that incorporates the dependency of mechanical properties of the expansive clay on its moisture content using the proposed effective stress-based constitutive model.

2 VOID RATIO CONSTITUTIVE SURFACE

The volume change behavior of expansive soils can be explained using the constitutive relationships that relate the deformation state variables (e.g. void ratio, e) to the stress state variables.

Two independent stress state approach using net normal stress and matric suction has been extensively used to explain the volume change and shear strength behavior of soils. In this approach, soil volume can change due to a change in mechanical stress and change in matric suction. Therefore, the constitutive relationships can be defined in the form of a constitutive surface in void ratio-net normal stress-matric suction space.

Several mathematical equations for description of void ratio constitutive surface have been proposed in the literature. Fredlund (1979) suggested that the void ratio constitutive surface for an unsaturated soil can be linearized over a wide range of stress changes using the logarithm of the stress state variables using the following equation:

$$e = a + b\log(\sigma - u_a) + c\log(u_a - u_w) \quad [1]$$

where a, b and c are fitting parameters.

Lloret and Alonso (1985) investigated several mathematical equations for description of the volume change constitutive surface of unsaturated soils under different loading conditions. By comparing best-fit to experimental data for different soil, they proposed the following equations:

$$e = a + b(\sigma - u_a) + c\log(u_a - u_w) + d(\sigma - u_a)\log(u_a - u_w) \quad [2]$$

$$e = a + b\log(\sigma - u_a) + c\log(u_a - u_w) + d\ log(\sigma - u_a)\log(u_a - u_w) \quad [3]$$

where a, b, c and d are fitting parameters. Eq. 3 was suggested as a more suitable equation for larger stress variation range.

Fredlund (2000) proposed the following equation for an overconsolidated soil:

$$e = a + b\ ln\left[1 + \left(\frac{\sigma}{c}\right)^2\right] + d\ ln\left[1 + \left(\frac{\sigma}{f}\right)^2\right] \quad [4]$$

where $a = a_1 + a_2(u_a - u_w)$, $b = b_1 + b_2\ln(u_a - u_w)$, $d = d_1 + d_2\ln(u_a - u_w)$ and a_1, a_2, b_1, b_2, c, d_1, d_2 and f are six fitting parameters.

Vu (2003) evaluated six different mathematical equations to define a void ratio constitutive surface for Regina clay. The following six-parameter equation was shown to provide an optimal constitutive surface:

$$e = a + b\ log\left[\frac{1 + c(\sigma - u_a) + d(u_a - u_w)}{1 + f(\sigma - u_a) + g((u_a - u_w)}\right] \quad [5]$$

where a, b, c, d, f and g are fitting parameters. Vu (2003) proposed a=1.183, b=-0.283, c=0.015, d=0.045, f=0, and g=0.00534 for Regina clay using laboratory test data obtained by Shuai (1996).

A constitutive surface can be simplified as a plane surface assuming linear relationship between the state variables. If linear relationship between void ratio and logarithm of net stress and logarithm of matric suction is considered, the plane constitutive surface can be explained by the following equation:

$$\frac{\sigma_m - u_a}{10^{\left(\frac{e-b}{a}\right)}} + \frac{u_a - u_w}{10^{\left(\frac{e-d}{c}\right)}} = 1 \quad [6]$$

where a, b, c and d are fitting parameters. The above equation describes a plane surface and therefore, suggests a linear relationship between net normal stress and matric suction at a constant void ratio. However, the constant void ratio curve for the void ratio constitutive surface does not necessarily have to be a straight line. Zhang (2004) indicated that a quadratic equation for constant void ratio curve can better describe the volume change behavior of soil and proposed the following mathematical expression for the void ratio constitutive surface:

$$(\sigma_m - u_a)10^{\left[y_1 - b_1\ln\left(\frac{a_1}{e - x_1} - 1\right)\right]} + (u_a - u_w)10^{\left[y_2 - b_2\ln\left(\frac{a_2}{e - x_2} - 1\right)\right]} = 1 \quad [7]$$

where a_1, a_2, b_1, b_2, x_1, x_2, y_1 and y_2 are fitting parameters. Zhang (2004) used the above equation to establish the constitutive surface for an expansive clay from a site at Arlington, Texas in the United States. The parameters proposed by Zheng (2004) to fit the equation to the available laboratory testing data are presented in Table 1.

Table 1. Fitting Parameters for Arlington expansive clay (Zhang 2004)

	a	b	x	Y
e vs $(\sigma - u_a)$	0.6641	-0.6811	3.3957	0.1731
e vs $(u_a - u_w)$	0.3604	-0.5073	2.7015	0.4740

Review of literature indicates that adequate mathematical representation of a surface describing the relationship between void ratio, matric suction and net normal stress requires three to eight empirical parameters to provide the best fit to the experimental data. In general, mathematical expressions with large number of fitting parameters provide better approximation of the experimental data. However, this may lead to loss of the physical interpretation of the parameters used in the equations (Lloret and Alonso, 1985).

3 SINGLE STRESS STATE VARIABLE RAMEWORK

The effective stress is known as a fundamental stress state variable that has been extensively used to calculate the soil strength and simulate the soil deformation behavior within the classical soil mechanics framework. Bishop (1954, 1959) extended the Terzaghi's effective stress to be used for unsaturated conditions:

$$\sigma' = (\sigma - u_a) - \chi(u_w - u_a) \quad [8]$$

where u_a is pore-air pressure, u_w is pore-water pressure, $\sigma - u_a$ is net total stress, and $u_w - u_a$ is matric suction. χ is scaling parameter depending on the soil saturation degree with the constraints of being zero when soil is completely dry and being unity when soil is fully saturated. The above single-valued effective stress equation has been shown to be inadequate for describing both volume change and shear strength behavior (Houston, 2019; Zhang and Lu, 2020). The early efforts to develop a single stress state framework for unsaturated soil conditions were later followed by two independent stress state variable approaches (Matyas and Radhakrishna 1968; Fredlund and Morgenstern 1977).

Despite its popularity, two stress state variable approach is subjected to a major practical limitation: It cannot be utilized within the context of classical soil mechanics that considers the effective stress as the single stress state variable governing the soil behavior (Lu et al. 2010). Moreover, smooth transition from unsaturated to saturated state in elasto-plastic models is another challenge in the use of the two independent stress state approach (Houston, 2019).

several researches have proposed modified forms of effective stress equation for unsaturated soil conditions (e.g., Khalili and Khabbaz 1998; Nuth and Laloui 2008; Lu and Griffiths 2004; Lu and Likos 2006; Lu et al. 2010; Alsherif and McCartney 2014; Manahiloh et al. 2016).

In this study, the single stress state approach by Lu et al. (2010) was utilized to calculate the effective stress. The evolution of the effective stress subject to changes in soil moisture was then used to anticipate the volumetric change behavior of the expansive soils.

Lu and Likos (2006) extended the pioneering work by Bishop (1954 and 1959), to define a new stress variable called suction stress (σ^s) within the context of Terzaghi's effective stress equation as following:

$$\sigma' = (\sigma - u_a) - \sigma^s \quad [9]$$

Lu et al. (2010) proposed a closed-form expression for suction stress for the full range of matric suction:

$$\sigma^s = -(u_a - u_w) \qquad (u_a - u_w) \leq 0 \quad [10]$$

$$\sigma^s = \frac{(u_a - u_w)}{(1 + [\alpha\,(u_a - u_w)]^n)^{(n-1)/n}} \qquad (u_a - u_w) \geq 0 \quad [11]$$

where α and n are the parameters used to define the soil water characteristic curve (SWCC) of a soil using van Genuchten's (1980) equation.

4 VOID RATIO - EFFECTIVE STRESS RELATIONSHIP USING REGRESSION ANALYSIS

The relationship of void ratio (e) versus logarithm of effective stress (σ') has been extensively utilized for saturated soils. For unsaturated soil conditions, laboratory testing to determine soil deformation behavior is normally carried out under controlled net total stress and controlled soil suction separately. In this study, the following equation was proposed to relate the variation of void ratio and change in mean effective stress ($\sigma_m{}'$):

$$e = a + bLn\left[1 + (\frac{\sigma_m{}'}{c})^2\right] \quad [12]$$

where a, b and c are fitting parameters. These constants were determined based on the available void ratio constitutive surface for two expansive soils that are described in Section 3 i.e., Texas expansive clay reported by Zhang (2004) and Regina expansive clay reported by Vu (2003).

MATLAB was employed for regression analysis using Multiple Linear Regression (MLR). The fitted curves for these two expansive clays are shown in Figure 1 and Figure 2. The regression parameters are presented in Table 2. The regression model performance was validated using several regression indices including R-Squared (R^2), mean square error (MSE) and root mean square error (RMSE). The obtained values for these indices are also presented in Table 2.

Table 2. Regression parameters for two expansive clays

	a	b	c	R^2	MSE	$RMSE$
Regina Clay	1.183	-0.032	8.475	0.960	1.79e-04	0.0134
Texas Clay	0.829	-0.038	45.697	0.993	4.76e-05	0.0069

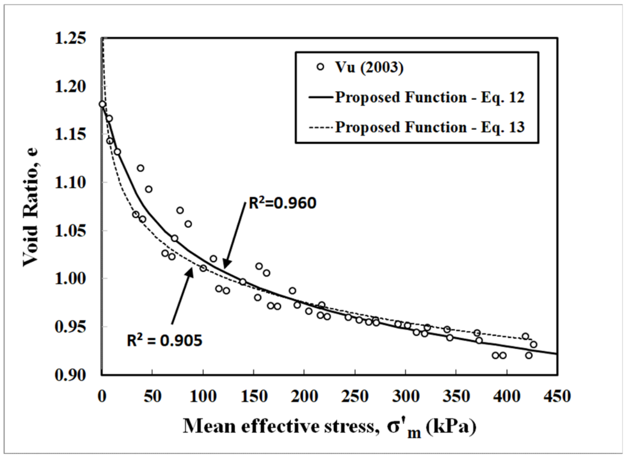

Figure 1. Fitted curve for Regina expansive clay reported by Vu (2003)

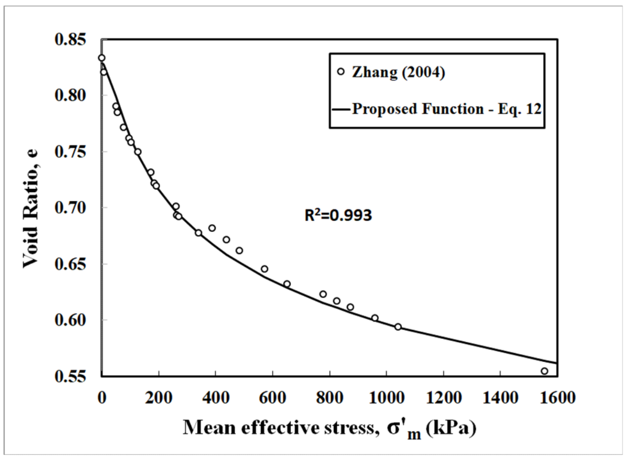

Figure 2. Fitted curve for Texas expansive clay reported by Zhang (2004)

In addition to Eq.12, the following equation with two fitting parameters was also examined to relate void ratio and mean effective stress for Regina expansive clay:

$$e = a + bLn[\sigma_m{}']$$ [13]

The fitted curve based on the above equation (a =1.52 and b =-0.052) was depicted in Figure 1. R-Squared (R^2) value was obtained approximately 0.906 which is less than R^2=0.960 for Eq.12.
The regression analysis for fitting the best curve to the void ratio vs mean effective stress determines three constants parameters used in Eq.12 and two parameters in Eq.13. However, calculation of effective stress also requires α and n that should be determined by fitting the van Genuchten (1980) equation to the soil water characteristic data. The computer program RETC (van Genuchten et al. 1991) was utilized to obtain the soil water characteristics parameters for the two expansive clays as presented in Table 3. SWCC curves considered for these two soils are presented in Figure 3.

Table 3. SWCC parameters α and n

	α (1/kPa)	n
Regina Expansive Clay	1.2350	1.10
Texas Expansive Clay	0.0029	1.27

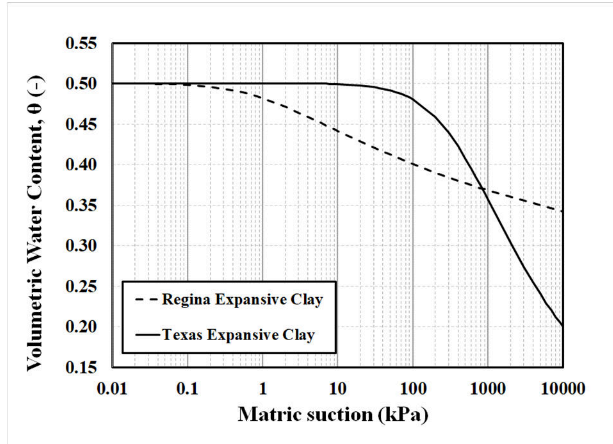

Figure 3. SWCC curves for two expansive clays in this study

5 CASE STUDY

5.1 Problem Statement

Figure 4 illustrates a 5 m thick layer of Regina expansive clay which is partially covered by a flexible pavement. The uncovered portion of the ground surface is subject to a constant infiltration rate of 1.73 mm/s. The initial matric suction is taken to be constant throughout the depth and equal to 400 kPa. This example was first simulated by Vu (2003) and has also been considered in later publications (e.g. Vu and Fredlund 2006, Adem and Vanapalli 2015). The variation of soil moisture conditions during 175 days was simulated using a variably saturated flow model followed by a one-dimensional deformation analysis.

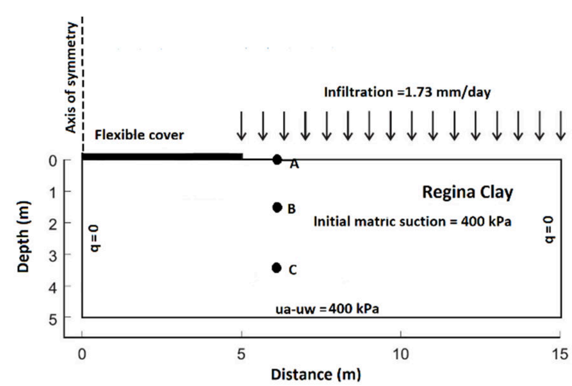

Figure 4. Geometry and hydraulic boundary conditions

5.2 Variably Saturated Flow Model

In this study, HYDRUS (2D/3D) version 3.04.0140 (Šimůnek et al. 2006) was utilized to simulate two-dimensional transient distribution of soil moisture. HYDRUS 2D is a finite element software that numerically solves the Richards' equation for the analysis of water flow in variably saturated soil.
The seepage finite-element model consisted of 10124 triangular elements. Finer mesh was generated near the unpaved portion of the top surfaces to provide appropriate resolution within the infiltration zone. At each time step, the pore pressure (matric suction) distribution affected by infiltration is updated by seepage analysis. The results of HYDRUS were then used to calculate the suction stress and effective stress field and the resultant soil deformations. The hydraulic boundary conditions for variably saturated seepage analysis are shown in Figure 4.

5.3 Deformation Analysis

As discussed in Section 3, the single stress state approach suggests that void ratio can be defined as function of effective stress that is identified as $e = g(\sigma_m')$. The evolution of mean effective stress with time will cause change in soil void ratio that can be approximated as follows:

$$\Delta e = \frac{\partial g}{\partial \sigma_m'} \Delta \sigma_m' \quad [14]$$

If a one-dimensional soil column is discretized into n sub-layers with thickness of h_i, its total vertical deformation for a time step (Δt) can be computed using the following equation:

$$\Delta h_t = \sum_{i=1}^{n} \Delta h_{i,t} = \sum_{i=1}^{n} \frac{\Delta e_{i,t}}{1 + e_{i,t-\Delta t}} h_i = \sum_{i=1}^{n} \frac{e_{i,t} - e_{i,t-\Delta t}}{1 + e_{i,t-\Delta t}} h_i \quad [15]$$

The above one-dimensional approach was utilized to estimate the cumulative heave evolution with time at nodes A, B and C as identified in Figure 4. The change in mean effective stress was calculated based on change in suction stress which was obtained from HYDRUS 2D model. The estimation heave with time was carried out by dividing the soil layer into 10 equal sub-layers of 0.5 m thickness. The at-rest lateral soil pressure coefficient (K_0) was calculated assuming the Poisson's ratio is equal to 0.4 as proposed by Vu and Fredlund (2006).

5.4 Results and Discussions

The variation of matric suction distribution was computed for 175 days of infiltration at the unpaved ground surface using HYDRUS 2D.
Figure 5 illustrate the distribution of suction head at day 175. The infiltration caused the matric suction to reduce from its initial value of 40.78 m (400 kPa) up to approximately 6 m at the vicinity of the unpaved ground surface. Figure 6 presents the matric suction profile with depth at 1m away from the covered surface at some sample different times. The matric suction profiles at smaller time step (0.1 day) were used as the primary input for deformation analysis.

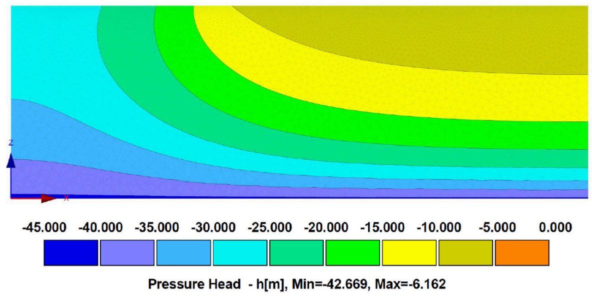

Figure 5. Suction head distribution at day 175

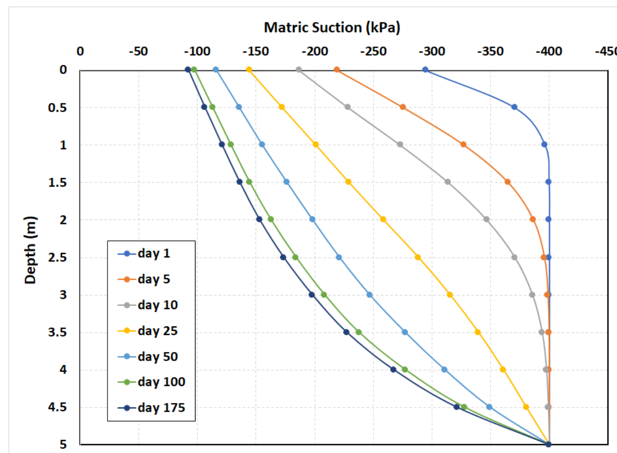

Figure 6. Matric suction profile with depth at different days

Figure 8 illustrates the comparison of predicted soil suction using the developed numerical model and the results obtained by Vu and Fredlund (2006) at the three locations (Nodes A, B, and C) over time. The observation nodes A, B and C are located 1m away from the paved area at depth 0, 1.5 m and 3.5 m, respectively. Although the anticipated matric suction at node C is approximately 20% less than what was reported by Vu and Fredlund (2006), the matric suction compares well at shallower depths (Nodes A and B).

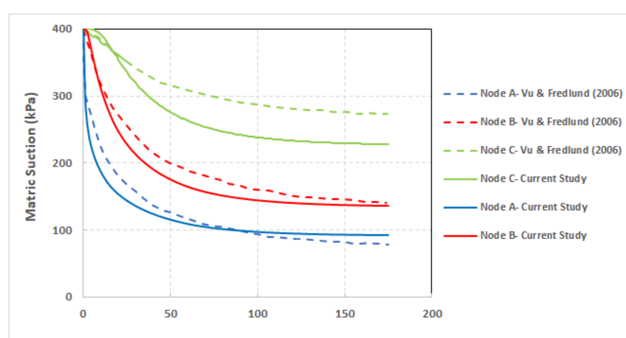

Figure 7. Matric suction development with time at Nodes A, B, C

Figure 8, Figure 9 and Figure 10 present the heave results, obtained from the deformation analysis proposed in this study, at Nodes A, B and C, respectively. Two sets of analyses were carried out using constitutive relationships represented by Eq.12 and Eq.13. Adem and Vanapalli (2013b) carried out a comparison study for estimating the heave for the same benchmark problem using three different one-dimensional deformation analysis methods: Hamberg (1985), Zhang and Briaud (2010) and Adem and Vanapalli (2013a). The heave prediction by these three methods as well as the reported heave values by Vu and Fredlund (2006) using two-dimensional analysis are also presented in these figures for comparison purposes.

Figure 8 indicates that the calculated heave at ground surface (Node A) from this study are in good agreement with Hamberg (1985) and Zhang and Briaud (2010). However, the obtained values are greater than Adem and Vanapalli (2013a) and Vu and Fredlund (2006). At deeper locations (i.e., Nodes B shown in Figure 9 and Node C shown in Figure 10) the anticipated variation of heave with time falls between the curves proposed by other studies. It should be noted that deformation analysis using Eq.12 resulted in greater heave values than Eq.13 at the investigated depths. This is mainly because Eq. 13 results in a constitutive curve with flatter slope at higher mean effective stress values.

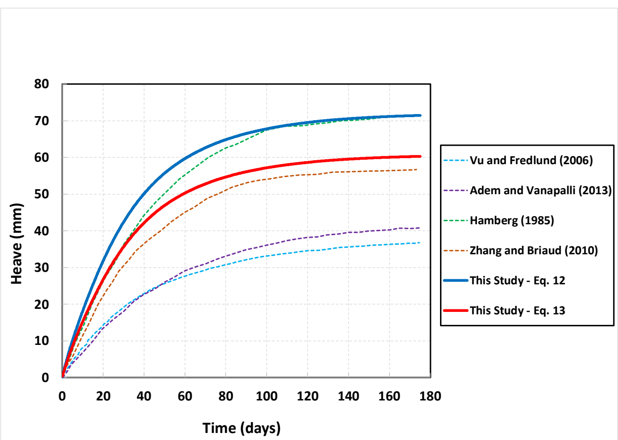

Figure 8. Variation of heave with time at Node A

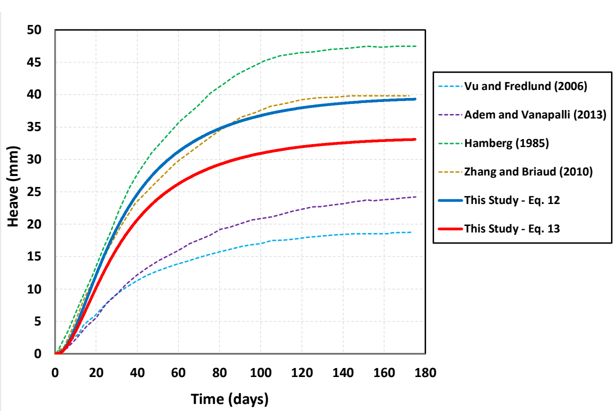

Figure 9. Variation of heave with time at Node B

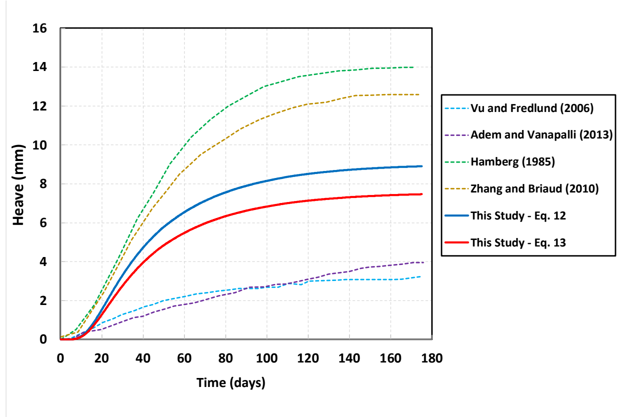

Figure 10. Variation of heave with time at Node C

6 CONCLUDING REMARKS

In this study, an effective stress-based approach was proposed to describe the void ratio as a function of effective stress for soils in unsaturated state. The proposed approach was used to fit single variable void ratio curves to the laboratory test data for two different expansive soils, i.e., Regina expansive clay and Texas expansive clay, under a range of mechanical stress and matric suction. Two different logarithm functions with two and three constant parameters were selected to examine the relationship between void ratio and mean effective stress in unsaturated conditions. Multiple Linear Regression (MLR) was employed to obtain the best fitted curves for constitutive relationship for two expansive clays. The regression analysis was validated using several regression indices.

The proposed constitutive relationship for Regina expansive clay was employed to model the problem of a partially covered soil layer subjected to a constant infiltration rate over 175 days. A two-dimensional unsaturated flow finite element analysis was utilized to calculate the matric suction profile at different times. The matric suction profiles along with soil water characteristic curve parameters, and the proposed constitutive function were then used as input in a one-dimensional deformation analysis. The findings were compared with the results obtained from several previous studies using different approaches.

In general, the results indicate the capability of the proposed single stress state (suction stress-based or effective stress-based) model in simulation of expansive soil deformations subject to varying moisture conditions over time using constitutive curves (void ratio vs effective stress) with fewer fitting parameters compared to conventional constitutive surfaces (void ratio vs net normal stress and matric suction). The validation of the developed model against various soil-atmosphere conditions and different expansive soil types over long term climate data considering climate change is part of the ongoing research.

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