Reliability bearing capacity analysis of footings seated on unreinforced and reinforced granular layers over undrained cohesive soil with spatial variability

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ABSTRACT
Strip footings located over cohesive soil deposits are most often constructed with a granular layer between the footing and the foundation soil to aid with construction and to disperse the footing loads over a wider area of the foundation soil surface. This paper investigates the case of a strip footing seated directly on an undrained cohesive clay deposit and the same footing placed on an unreinforced and geosynthetic (geogrid) reinforced granular layer. The comparisons are made using a probabilistic framework and considering the influence of isotropic and anisotropic spatial variability of the clay foundation undrained shear strength and modulus. Probability of failure is defined as the probability that the footing bearing capacity is less than the factored design deterministic value. The numerical modelling uses the random finite element method (RFEM) with random fields generated using the local averaging subdivision method (LAS). The mean values for bearing capacity increase in the order of no granular layer, unreinforced granular layer and reinforced granular layer at each value of spatial correlation length. However, the probability of failure is shown to be greater for the homogenous random soil foundation case (i.e., less safe for design) than for the same foundation soil with spatially isotropic properties.

RÉSUMÉ
Les semelles en bande situées sur des dépôts de sol cohésifs sont le plus souvent construites avec une couche granulaire entre la semelle et le sol de fondation pour faciliter la construction et disperser les charges de semelle sur une plus grande surface de la surface du sol de fondation. Cet article étudie le cas d'une semelle de bande assise directement sur un dépôt d'argile cohésive non drainée et de la même semelle placée sur une couche granulaire non renforcée et renforcée par géogrilles. Les comparaisons sont faites à l'aide d'un cadre probabiliste et en tenant compte de l'influence de la variabilité spatiale isotrope et anisotrope de la résistance et du module de cisaillement non drainés de la fondation en argile. La probabilité de défaillance est définie comme la probabilité que la capacité portante de base soit inférieure à la valeur déterministe de conception factorisée. La modélisation numérique utilise la méthode des éléments finis aléatoires (RFEM) avec des champs aléatoires générés à l'aide de la méthode de subdivision de moyenne locale (LAS). Les valeurs moyennes de la capacité portante augmentent de l'ordre de l'absence de couche granulaire, de couche granulaire non renforcée et de couche granulaire renforcée à chaque valeur de longueur de corrélation spatiale. Cependant, la probabilité de défaillance s'avère plus grande pour le cas homogène de fondation aléatoire du sol (c.-à-d. moins sécuritaire pour la conception) que pour le même sol de fondation ayant des propriétés isotropes spatiales.

1 INTRODUCTION

Ultimate bearing capacity analysis and design of shallow footings is a classical problem in geotechnical engineering. Broadly defined, the ultimate bearing capacity of a footing is the maximum load (or pressure) that can be applied to the base of the footing without causing the underlying soil to fail. Solutions for the bearing capacity problem can be found in geotechnical engineering text books and design manuals. In North American geotechnical foundation engineering practice, the Terzaghi solutions and equations by Meyerhof are most popular (e.g., CFEM 2006; CSA 2019; AASHTO 2020). These solutions are based on the concept of limit equilibrium with soil strength described by deterministic Mohr-Coulomb strength parameters.

The classical solutions noted above apply to single soil layers. In practice, footings are typically placed on or in a granular layer that in turn is seated on a natural soil deposit or perhaps a fill of lower quality. Of particular interest to the current study is the case of a granular layer overlying an undrained cohesive clay soil. Bearing capacity solutions for double soil layers have been developed by Hanna and Meyerhof (1980) and Kenny and Andrawes (1997), and investigated using more advanced numerical methods by Burd and Frydman (1997), Salimi et al. (2018) and Michalowski and Shi (1995), amongst others.

One strategy to improve the ultimate bearing capacity of a granular layer seated on a weaker stratum is to place a layer of geosynthetic reinforcement at the base of the granular layer (Figure 1). Analytical solutions for the ultimate bearing capacity of footings using this reinforcement technique with an undrained clay foundation can be found in the work of Saha Roy and Deb (2017) as one example.

Both unreinforced and reinforced foundation solutions in these earlier works assume that the soil in each layer is homogenous and isotropic. These assumptions have the advantage of allowing tractable closed-form solutions to be

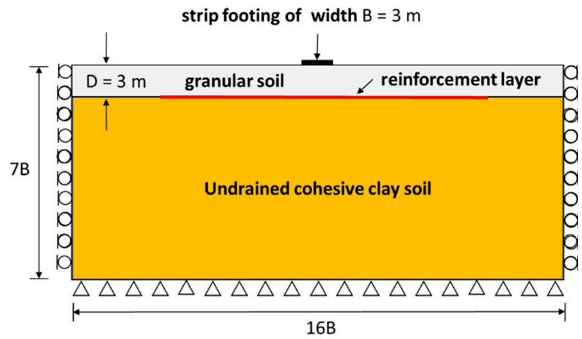

Figure 1. Strip footing seated on reinforced granular base over undrained cohesive clay soil foundation.

used to compute the ultimate bearing capacity of the footing. However, all soil materials have random and spatial variability to varying degrees. The influence of uncertainty in soil properties on the ultimate bearing capacity of a footing seated directly on a clay foundation has received attention by Griffiths and Fenton (2001), Fenton and Griffiths (2002), and Griffiths et al. (2002). A practical consequence of variability in soil strength properties is that two footings that have the same deterministic factor of safety cannot be expected to have the same margin of safety expressed in probabilistic terms.

2 OBJECTIVES

The main objective of this study is to explore the influence of random and spatial variability of the undrained shear strength of an undrained cohesive soil foundation on the probability of bearing capacity failure of a rigid strip footing. The foundation conditions examined are the footing sitting directly on the cohesive soil layer, and the same footing seated on a granular layer with and without a geosynthetic reinforcement layer at the interface between the granular layer and clay foundation (Figure 1).

3 PRIOR RELATED WORK ON BEARING CAPACITY OF FOOTINGS ON UNDRAINED COHESIVE SOIL

The earliest work on the influence of random soil fields on bearing capacity of idealized unreinforced simple footings on undrained cohesive soil foundations can be traced to the contributions of Griffiths and Fenton (2001) and Fenton and Griffiths (2002). They used the random finite element method (RFEM) together with the local averaging subdivision method (LAS) implemented in the open-source FEM code (*rbear2d*) described by Fenton and Griffiths (2008). Only a single layer of purely cohesive soil was considered and the footing was located at the top of the soil domain. They showed that spatial variation of the undrained shear strength could affect the probability of bearing capacity failure of a simple footing resting on a cohesive soil layer. Since this initial work, additional RFEM studies using unreinforced soils have been reported by Fenton and Griffiths (2008) and later Cho and Park (2010) who found that cross correlation between the strength parameters can influence the probability of failure of a footing. They also demonstrated that probability of bearing capacity failure for the same deterministic factor of safety changes with spatial correlation length of soil shear strength. Luo and Bathurst (2017) extended the footing problem to the case of a footing placed in proximity to the crest of a simple slope with cohesive soil having random and spatial variability. They observed similar trends to the earlier work cited above. Fenton and Griffiths (2003) carried out RFEM of footings on single layers of cohesive-frictional (c-ϕ) soils. These soil conditions are beyond the scope of the current study.

An important lesson from these prior studies is that the bearing capacity of shallow footings with spatially varying soil parameters can have lower bearing capacity (on average) compared to the same soils with only random soil property variability.

4 NUMERICAL MODELLING

4.1 RFEM code

The random finite element method is a probabilistic approach that combines the finite element method, random field theory, and Monte Carlo simulation. This technique generates randomly assigned properties for each discretized mesh in a conventional finite element domain. The open source *rbear2d* code for 2-D shallow foundation stochastic bearing capacity analysis by Fenton and Griffiths (2008) was modified by the authors to investigate unreinforced and reinforced granular layers. The important modifications made to the program to carry out the scope of work in this study are described below:

- The original code is limited to a single random field with each property described by mean (μ), standard deviation, (σ), and spatial variability (Θ). The ability to generate multi-random field zones with and without random and spatially variable properties was added. In this study, a granular top layer was considered to be an engineered soil with constant properties, while the underlying natural deposit of cohesive soil was assigned random and spatial undrained shear strength and elastic modulus.
- 1-D quadratic bar elements, representing geosynthetic reinforcement layers in the soil were added. The main contribution of a geosynthetic layer(s) in a reinforced footing scenario is to add stiffness and strength to the foundation system. The original code was modified to include one or more zero thickness reinforcement layers modelled as 3-node bar (rod) stiffness elements with user-defined parameters. However, the scope of the current study is restricted to a single layer of reinforcement placed at the base of the granular layer. This bar element is a simplified form of a general beam element having 6 degrees of freedom with no bending resistance. This element was added to the corresponding degrees of freedom in the global stiffness matrix. The user has full control over the geometry of the reinforcement inclusion (e.g., length,

elevation and inclination, although the latter was not investigated in this study).

- The original code produces a random field with cells matching the finite element mesh size. The LAS module was changed to allow the user to produce random fields with user-defined subdivisions. This is a valuable modification that allows the contribution of random field resolution to numerical outcomes to be isolated from the size of the finite element mesh.

4.2 Random field simulation using LAS method

Random field models are used to describe stationary and spatial variability of soil properties in the ground. The inherent variability of stationary and lognormally distributed random field parameters can be described by the mean, the variance (or coefficient of variation (COV)), and the scale of fluctuation (e.g., spatial correlation length) (Vanmarcke 1984). The definition of mean and variance of random data are familiar to geotechnical engineers. Spatial correlation length (Θ) quantifies dependency of variables (such as soil shear strength, unit weight, elastic modulus) with respect to distance. Shorter correlation lengths mean that higher spatial frequencies dominate and greater variances of data exist, while longer correlation lengths correspond to smoother variations that occur over longer distances. In cases of very long correlation lengths with respect to the domain of interest, spatial variation can be ignored and the problem reduced to the single random variability case in which only one value of the random variable is assigned to the entire domain for each realization.

Anisotropic spatial variability in the current study refers to conditions where spatial variability is different in vertical and horizontal directions denoted by Θ_y and Θ_x, respectively. For the domain scales that are important to the footing problem, $\Theta_y < \Theta_x$.

Various techniques are available to simulate random fields of soil properties. In this study the local averaging subdivision method (LAS) (Fenton and Vanmarke 1990) is used. The reader is directed to Fenton and Griffiths (2008) for details and its implementation in the RFEM code used in the current study.

4.3 Numerical model

The general problem domain for the footing seated on a reinforced granular layer is shown in Figure 1. The footing width B = 3 m and thickness of the granular layer D = 3 m are shown in the figure. The domain dimensions of 16B and 7B boundaries were selected to be as large as possible to minimize boundary effects while avoiding excessive execution times using the RFEM code.

The cohesive (natural) soil layer was assigned an undrained shear strength s_u = 100 kPa (stiff to very stiff clay), undrained friction angle ϕ_u = 0, Poisson's ratio = 0.495 and undrained Young's modulus E_u = 10 MPa. The top soil layer was assumed to be a high quality granular soil (e.g., gravel) with peak friction angle ϕ = 45° and a small cohesive strength component c = 5 kPa. The elastic modulus and Poisson's ratio for the granular layer were taken as 100 MPa and 0.3, respectively. Soil properties used in analyses are summarized in Table 1.

Table 1. Soil parameters used in deterministic and probabilistic analyses.

Parameter	Mean value	COV
Granular layer		
ϕ	45°	0
c	5 kPa	0
ψ (dilatancy angle)	0°	0
E	100 MPa	0
Foundation		
s_u	100 kPa	0.5
ϕ_u	0	0
E_u	10 MPa	0.5

Deterministic parametric analyses were carried out using the commercial finite element program Sigma/W (GeoSlope Ltd. 2018) and a purely linear-elastic plastic domain to compute deterministic bearing capacity values for the three foundation cases. The footing was advanced in small displacement increments. The bearing capacity of the footing in the deterministic bearing capacity analyses was taken as the lesser of the footing pressure after a footing settlement of 0.05B (i.e., 0.15 m), or the maximum load before numerical instability. The reference deterministic bearing capacity values are summarized in Table 2.

Analyses using Sigma/W also confirmed that vertical stresses developed at the footing were dissipated to about 80% of the contact pressure within 5B below the footing. Sensitivity analyses using Sigma/W using the same mesh size and problem domain showed that the deterministic factor of safety against bearing capacity failure (as defined below) was insensitive to E_u = 10, 20 and 100 MPa for a ratio of D/B = 1 that was used in this study.

For probabilistic analyses in this study, s_u and E_u are random values with mean and COV values shown in Table 1. Both random variables were assumed to be lognormally distributed to avoid sampling negative values during Monte Carlo (MC) simulations. The granular layer properties were taken as deterministic. This is a reasonable assumption because these layers are most often engineered granular fills that are placed and compacted to satisfy a construction

Table 2. Deterministic bearing capacity values.

Case	Bearing capacity* (kPa)
No granular soil	514
Granular soil	765
Reinforced granular soil **	806

* lesser of the footing pressure after a footing settlement of 0.05B or the maximum load before numerical instability
** J = 5000 kN/m

specification. Consequently, random and spatial variability is negligible, particularly when compared to natural soil deposits.

All numerical analyses were carried out with dilatancy angle ψ = 0 (non-associated flow rule). This assumption is judged to be conservative for design since deterministic sensitivity analyses using the computer program Sigma/W showed that the footing response was less stiff using this assumption as opposed to $\psi = \phi$ (associated flow rule) which is consistent with Bolton (1986) and Payan et al. (2022).

For the RFEM analyses, the finite element mesh size of 0.025 m was adopted for all cases. Eight-node quadrilateral elements were used with a reduced four-point Gauss integration rule to calculate the element stiffness matrices.

For each RFEM realization in this investigation, the footing with load Q = Q(deterministic)/FS was placed at the top boundary. Failure was assumed if numerical convergence was not satisfied or surface settlement reached 0.05B. Most often in the analyses to follow, the first criterion controlled bearing capacity outcomes. The reason for this is that the soil in this investigation was very stiff with high shear strength which resulted in collapse (shear failure) before the settlement criterion was achieved.

The reinforcement layer was assumed to be continuous in the plane-strain direction of the problem domain. The reinforcement layer was modelled with tension-only elastic bar elements using the approach of Luo et al. (2016). For the base case in the analyses to follow, the elastic modulus (E) and bar thickness (b) were selected to give a stiffness value J = E$\times$b = 5000 kN/m. This stiffness is at the high end for geotextile and geogrid reinforcement products used in soil reinforcement applications according to a database compiled by Bathurst and Naftchali (2021).

The reinforcement layer was assumed to be fully bonded to the granular soil. This is a reasonable assumption for geogrid reinforcement products which have apertures and are embedded in granular soils. The fully bonded assumption has led to numerical predictions for MSE walls constructed with granular soil and geogrid reinforcement that were judged to be in satisfactory agreement with measured performance (Hatami and Bathurst 2005, 2006; Huang et al. 2009).

4.4 Probability of failure

Probabilities of bearing capacity failure reported later are taken with respect to the deterministic ultimate bearing capacity of the footing using the same problem configuration and mean soil properties (Table 1). The factor of safety (FS) for each simulation with random or spatial soil properties is the ratio of the maximum bearing pressure achieved in the RFEM simulation to the value of the deterministic bearing capacity (Table 2).

In the figures to follow the probability of failure (P_f) was computed as the percent of realizations which are exceeded by the deterministic value divided by the factor of safety. In previous related work, the deterministic value divided by FS is called the design bearing capacity.

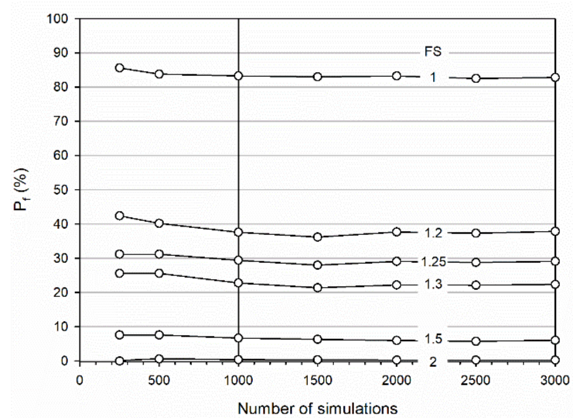

Figure 2. Sensitivity of probability failure (P_f) to number of MC simulations for FS = 1 to 2 and footing placed directly on cohesive soil foundation with random variables E_u and s_u with COV = 0.5.

4.5 Number of simulations

A major part of the computational expense using the RFEM approach is the requirement to carry out a large number of realizations (simulations). For the numerical computations presented later, a total of n = 1000 MC simulations was used. To investigate if this number was satisfactory, simulations for the case of the (unreinforced) strip footing of width B sitting directly on the clay layer were carried out with different footing loads. Each simulation was carried out with a homogenous clay foundation with random values of E_u and s_u sampled from lognormal distributions with mean and COV values shown in Table 1. Thus, each random field is equivalent to $\Theta_y = \Theta_x$ = infinity. Each footing load (Q) was computed as the Prandtl load divided by factor of safety (FS) (i.e., Q = 5.14 $s_u \times$ B/FS). The range of load in this investigation corresponds to FS from 1 to 3. The properties for the clay in the probabilistic (RFEM) analyses are the same as those in Table 1. The results of simulations showed that the computed probabilities of failure were the same for n ≥ 1000 from a practical point of view (Figure 2). The data plots in this figure also show that as FS increases, the probability of failure decreases, which is expected; furthermore, P_f was practically 0 for FS ≥ 1.5.

4.6 Probabilistic bearing capacity analysis of footing placed directly on cohesive soil

In order to isolate the influence of spatial correlation length on footing bearing capacity and to confirm a worst correlation length for undrained shear strength, a set of analyses were run without a granular seating layer as in the previous section. The minimum correlation length (Θ) was taken equal to the FEM mesh size (0.025 m). The recommended minimum Θ value is twice the FE mesh size according to the recommendations of Huang and Griffiths (2015). However, simulations with correlation lengths matching the FE mesh (element) size in the current

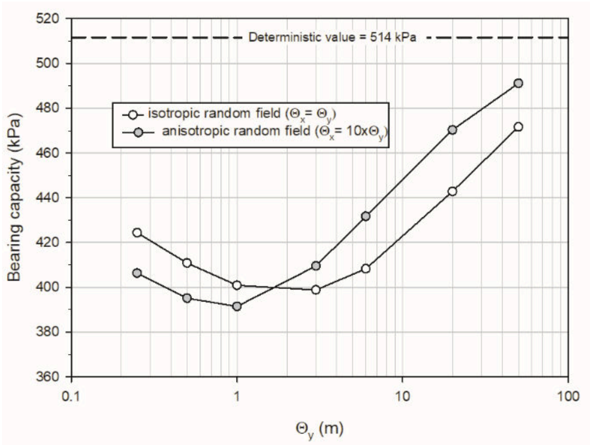

Figure 3. Mean bearing capacity versus correlation length (Θ) for isotropic and anisotropic spatial variability for case of footing seated directly on clay foundation.

investigation gave similar results with ratios of correlation length to FE mesh equal to 2.

Mean bearing capacities versus correlation length are plotted in Figure 3. Results for isotropic and anisotropic spatial variability cases are shown. For the anisotropic case, $\Theta_x = 10\times\Theta_y$ was used since this ratio of vertical to horizontal correlation length captures the predominant layering in the vertical direction. In the limit of $\Theta_y \rightarrow \infty$, isotropic and anisotropic foundation conditions will converge to the case of a random homogeneous foundation and the mean value of bearing capacity will approach the deterministic value.

An important observation from Figure 3 is that the mean bearing capacity from 1000 realizations with spatial variability is always less than the deterministic value of 514 kPa. This outcome is the result of the critical slip surface in each FEM realization that seeks the weakest path through the soil (Griffiths and Fenton 2001; Fenton and Griffiths 2003). For the isotropic cases, the worst-case correlation length value is in the vicinity of 3 m, which is the footing width. This observation is in agreement with the results of Fenton and Griffiths (2002). For the anisotropic case, the most critical value is 1 m. However, as a practical

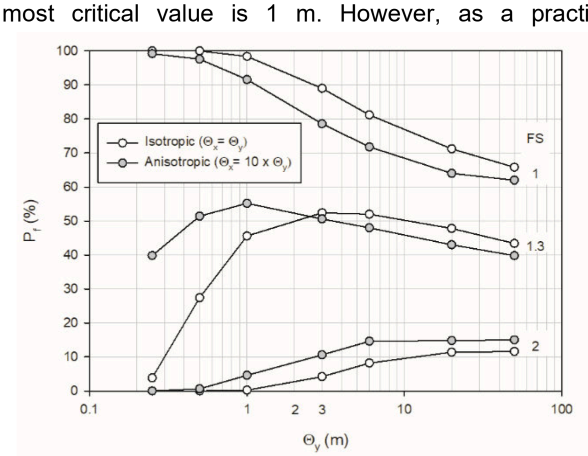

Figure 4. Probability of bearing capacity failure for footing on cohesive soil versus isotropic and (vertical) anisotropic spatial variability (correlation length) and different reference deterministic factors of safety.

observation considering the wide range of Θ values investigated and the range of bearing pressures, it can be said that worst case scenarios are for spatial correlation lengths in the vicinity of the footing width.

Figure 4 shows plots of P_f for different spatial variability conditions. As expected, as the factor of safety increases (i.e., the design bearing capacity decreases) the probability of failure decreases for the same combination of Θ_x and Θ_y correlation lengths. However, at FS in the vicinity of 1 the probability of failure is greater for the isotropic case than for the anisotropic case. In the vicinity of FS = 1.3 there is a reversal of this trend for $\Theta_y \leq 3$.

4.7 Probabilistic bearing capacity analysis of footing placed directly on granular layer over cohesive soil foundation

Margins of safety against bearing capacity failure of a footing on a cohesive soil foundation will increase if a granular layer is located between the footing and the foundation. For example, the deterministic maximum bearing capacity for D/B = 1, $\phi = 45°$ and the other mean soil parameter values in Table 1, is 765 kPa. This value was found by incrementally increasing the footing load until collapse as described in Section 4.3 using the Slope/W program. The bearing capacity for the footing seated on the granular layer is roughly 50% larger than the value of 514 kPa for the footing seated directly on the cohesive soil foundation.

Figure 5 shows that the trends in probability of failure are the same as in Figure 4 but lower in magnitude for cases with FS ≤ 1.3. The critical correlation length can be seen to migrate to the right (become larger) for the isotropic and anisotropic cases for FS = 1.3 when compared to Figure 4. An explanation for this trend is load spreading through the depth of the granular layer.

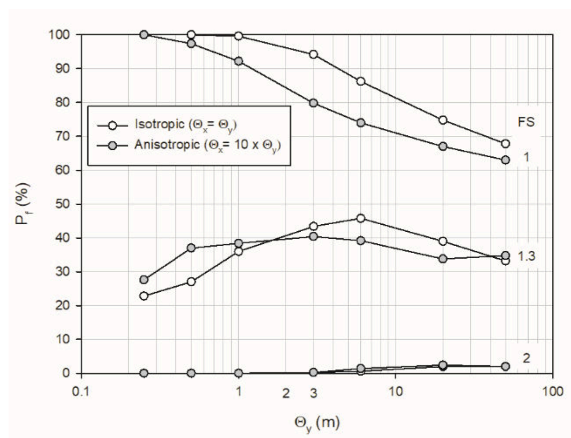

Figure 5. Probability of bearing capacity failure for footing on granular layer overlying cohesive soil versus isotropic and (vertical) anisotropic spatial variability (correlation length) and different reference deterministic factors of safety.

4.8 Probabilistic bearing capacity analysis of footing placed directly on reinforced granular layer over cohesive soil foundation

Deterministic modelling and probabilistic analyses for the cases described in the previous section were repeated with a single layer of geosynthetic reinforcement placed at the base of the granular layer. The reinforcement layer was assigned a linear elastic stiffness of 5000 kN/m. A check on sensitivity of probability of failure outcomes to number of MC simulations was performed and it was confirmed again that 1000 realizations was sufficient.

The maximum bearing capacity of the footing was 806 kPa which is 5% greater than the value of 765 kPa for the same footing condition and no reinforcement (Table 2). The increase in bearing capacity can be judged to be minor. A review of the literature shows that the benefit of a reinforcement layer will decrease with increasing foundation strength (and stiffness). Therefore, compared to the present study, the benefit of a layer of reinforcement can be expected to increase for foundation soils with s_u < 100 kPa used in the current study (e.g., Love et al. 1987).

Figure 6 shows values of computed P_f with isotropic and anisotropic spatial correlation length and factor of safety. The computed values for P_f are always lower than for the case without a reinforcement layer (see Figure 5). The critical correlation length increases for the isotropic and anisotropic cases for FS = 1.3 when compared to Figure 5. This can be explained by the further load spreading due the presence of the reinforcement layer.

Reinforcement strains were shown to be in the vicinity of 0.4% and not greater than 0.5% and thus within the linear-elastic strain range of typical geosynthetic soil reinforcement materials. In fact, reinforcement strains with these magnitudes may be difficult to measure in the laboratory (Allen and Bathurst 2019) or in the field (Bathurst et al. 2002). Reinforcement strains may be expected to increase for weaker (softer) cohesive soil foundations. However, such an investigation is beyond the scope of this preliminary study.

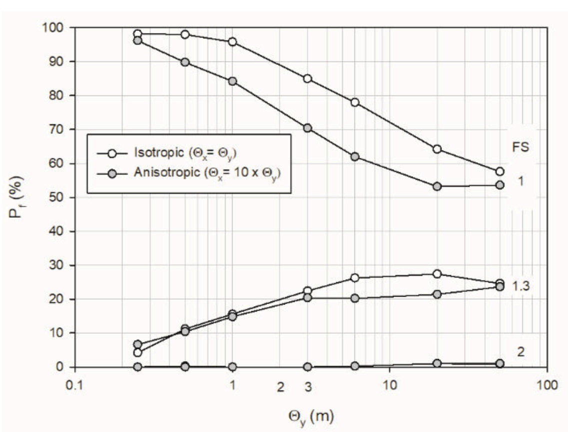

Figure 6. Probability of bearing capacity failure for footing on reinforced granular layer overlying cohesive soil versus isotropic and (vertical) anisotropic spatial variability (correlation length) and different reference deterministic factors of safety.

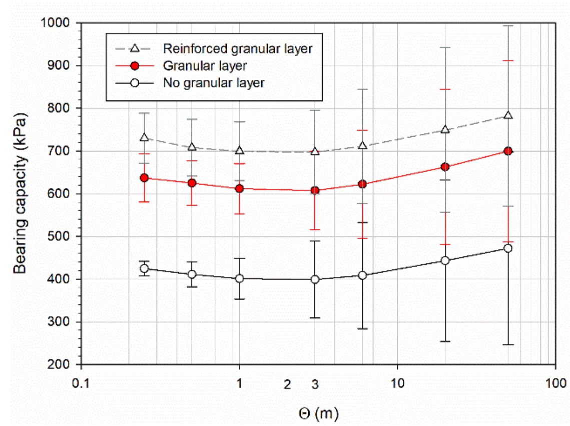

Figure 7. Comparison of mean and spread in computed bearing capacity for different foundation cases and range of isotropic correlation length. Range bars are ± two standard deviations.

4.9 Comparison of three foundation cases

Figure 7 shows a comparison of mean computed bearing capacity for the three foundation cases with isotropic spatial variability (i.e., $\Theta = \Theta_x = \Theta_y$). As expected, the mean values for bearing capacity increase in the order of no granular layer, unreinforced granular layer and reinforced granular layer at each value of Θ. Also shown in this figure are the spreads in computed values described by the ± two standard deviation range bars. These data show that the spread in computed bearing capacity increases with increasing Θ. In the limit of $\Theta \to \infty$ (i.e., random homogenous foundation soil) the probability of failure (P_f) for the homogenous case is always greater than that for the same foundation soil with spatially isotropic soil strength and stiffness. A practical outcome from this observation is that if the deterministic solution for bearing capacity is known, then the assumption of a homogenous random soil foundation will always give a greater probability of failure (i.e., less safe for design) than for the same soil with spatially isotropic properties.

5 CONCLUSIONS

This paper explores in a preliminary way the influence of random and spatial variability of the undrained shear strength of a purely cohesive soil foundation on the probability of bearing capacity failure of a footing seated on different foundations. The foundation conditions examined are the footing sitting directly on a cohesive soil layer, and the same footing seated on an unreinforced granular layer, and a reinforced granular layer with one geosynthetic reinforcement layer. Foundation soils were examined for the case of random, isotropic and anisotropic soil strength and stiffness. Some of the major findings from this paper are as follows:

- The mean values for bearing capacity increase in the order of no granular layer, unreinforced granular layer and reinforced granular layer at each value of spatial correlation length.

- The critical correlation length corresponding to the highest probability of failure for FS = 1.3 was observed to increase with increasing isotropic spatial variability. This is understood to be the result of load spreading which is greatest for the reinforced granular layer case.
- However, if the deterministic solution for bearing capacity is known, then the assumption of a homogenous random soil foundation will always give a greater probability of failure (i.e., less safe for design) than for the same soil with spatially isotropic properties.

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REFERENCES

AASHTO 2020. LRFD Bridge Design Specifications, 9th Ed., American Association of State Highway and Transportation Officials (AASHTO), Washington, DC.

Allen, T.M. and Bathurst, R.J. 2019. Geosynthetic reinforcement stiffness characterization for MSE wall design. *Geosynthetics International* 26(6): 592-610.

Bathurst, R.J., Allen, T.M. and Walters, D.L. 2002. Short-term strain and deformation behavior of geosynthetic walls at working stress conditions. *Geosynthetics International* 9(5-6): 451-482.

Bathurst, R.J. and Naftchali, F.M. 2021. Geosynthetic reinforcement stiffness for analytical and numerical modelling of reinforced soil structures. *Geotextiles and Geomembranes* 49: 921-940.

Bolton, M.D. 1986. Strength and dilatancy of sands. *Geotechnique* 36(1): 65-78.

Burd, H.J. and Frydman, S. 1997. Bearing capacity of plane-strain footings on layered soils. *Canadian Geotechnical Journal* 34(2): 241-253.

Canadian Foundation Engineering Manual. 2006. 4th ed. Canadian Geotechnical Society. Richmond, BC, Canada.

Canadian Standards Association (CSA). 2019. *Canadian Highway Bridge Design Code*. CAN/CSA-S6-19, Mississauga, Ontario, Canada.

Cho, S.E. and Park, H.C. 2010. Effect of spatial variability of cross-correlated soil properties on bearing capacity of strip footing. *International Journal for Numerical and Analytical Methods in Geomechanics* 34(1): 1-26.

Fenton, G.A. and Griffiths, D.V. 2002. Probabilistic foundation settlement on spatially random soil. *ASCE Journal of Geotechnical and Geoenvironmental Engineering* 128(5): 381-390.

Fenton, G.A. and Griffiths, D.V. 2003. Bearing-capacity prediction of spatially random c-ϕ soils. *Canadian Geotechnical Journal* 40(1): 54-65.

Fenton, A.G. and Griffiths, D.V. 2008. *Risk Assessment in Geotechnical Engineering*. John Wiley and Sons. Inc., New Jersey, USA.

Fenton, G.A. and Vanmarcke, E.H. 1990. Simulation of random fields via local average subdivision. *Journal of Engineering Mechanics* 116(8): 1733-1749.

GeoSlope Ltd. 2018. *Stress-deformation modeling with Sigma/W*. GeoSlope International Ltd., Calgary, Canada.

Griffiths, D.V. and Fenton, G.A. 2001. Bearing capacity of spatially random soil: the undrained clay Prandtl problem revisited. *Geotechnique* 51(4): 351-359.

Griffiths, D.V., Fenton, G.A. and Manoharan, N. 2002. Bearing capacity of rough rigid strip footing on cohesive soil: probabilistic study. *ASCE Journal of Geotechnical and Geoenvironmental Engineering* 128(9): 743-755.

Hanna, A.M. and Meyerhof, G.G. 1980. Design charts for ultimate bearing capacity of foundations on sand overlying soft clay. *Canadian Geotechnical Journal* 17(2): 300-303.

Hatami, K. and Bathurst, R.J. 2005. Development and verification of a numerical model for the analysis of geosynthetic-reinforced soil segmental walls under working stress conditions. *Canadian Geotechnical Journal* 42(4): 1066-1085.

Hatami, K. and Bathurst, R.J. 2006. Numerical model for reinforced soil segmental walls under surcharge loading. *ASCE Journal of Geotechnical and Geoenvironmental Engineering* 132(6): 673-684.

Huang, B., Bathurst, R.J. and Hatami, K. 2009. Numerical study of reinforced soil segmental walls using three different constitutive soil models. *ASCE Journal of Geotechnical and Geoenvironmental Engineering* 135(10): 1486-1498.

Huang, J. and Griffiths, D.V. 2015. Determining an appropriate finite element size for modelling the strength of undrained random soils. *Computers and Geotechnics* 69: 506-513.

Kenny, M.J. and Andrawes, K.Z. 1997. The bearing capacity of footings on a sand layer overlying soft clay. *Geotechnique* 47(2): 339-345.

Love, J.P., Burd, H.J., Milligan, G.W.E. and Houlsby, G.T. 1987. Analytical and model studies of reinforcement of a layer of granular fill on a soft clay subgrade. *Canadian Geotechnical Journal* 24(4): 611-622.

Luo, N. and Bathurst, R.J. 2017. Reliability bearing capacity analysis of footings on cohesive soil slopes using RFEM. *Computers and Geotechnics* 89: 203-212.

Luo, N., Bathurst, R.J. and Javankhoshdel, S. 2016. Probabilistic stability analysis of simple reinforced slopes by finite element method. *Computers and Geotechnics* 77: 45-55.

Payan, M., Fathipour, H., Hosseini, M., Jamshidi Chenari, R. and Shiau, J.S. 2022. Lower bound finite element limit analysis of geo-structures with non-associated flow rule. *Computers and Geotechnics* 147: 104803.

Michalowski, R.L. and Shi, L. 1995. Bearing capacity of footings over two-layer foundation soils. *Journal of Geotechnical Engineering* 121(5): 421-428.

Saha Roy, S. and Deb, K. 2017. Effects of aspect ratio of footings on bearing capacity for geogrid-reinforced sand over soft soil. *Geosynthetics International* 24(4): 362-382.

Salimi Eshkevari, S., Abbo, A.J. and Kouretzis, G. 2019. Bearing capacity of strip footings on sand over clay. *Canadian Geotechnical Journal* 56(5): 699-709.

Vanmarcke, E.H. 1984. *Random fields: Analysis and synthesis.* MIT Press, Cambridge, Mass.