Comparison of Modelling Slope Failure using Shear Strength Reduction (SSR) Procedures and an Informed Multi-Stage Approach

Liam Kelly, D. Jean Hutchinson & Mark Diederichs
Geological Sciences & Geological Engineering – Queen's University, Kingston, Ontario, Canada

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ABSTRACT
Geotechnical engineers commonly rely on standard geotechnical Finite Element Method (FEM) software with built-in Shear Strength Reduction (SSR) procedures to compute the Factor of Safety (FoS) for slope stability problems. Some built-in SSR procedures, such as in Rocscience's RS2, solve SSR trials as individual and independent analyses, not considering previously computed trials. This paper compares the results obtained using RS2's built-in SSR procedures to a multi-stage approach where strength parameters are reduced manually, and the previously computed trial solutions inform the subsequent SSR trial models. Homogeneous and multi-material elastic-perfectly plastic slope models are developed to compare the results for FoS, run time, and nodal displacement patterns. The "informed" multi-stage models result in faster trial solution convergence and overall model computation time compared to traditional SSR analyses, especially for fine FEM meshes and models with stricter convergence tolerance. Additionally, the multi-staged strength reduction models result in smoother nodal displacements than SSR trial solutions. The staged strength reduction models result in Factors of Safety equal to or less than the built-in SSR procedure for all slope models developed.

RÉSUMÉ
Les ingénieurs en géotechnique se fient généralement sur des logiciels standards de la méthode des éléments finis (MEF) qui utilisent des procédures intégrées de réduction de la résistance au cisaillement (SSR) pour calculer le facteur de sécurité (FoS) des problèmes de stabilité des pentes. Certaines procédures SSR intégrées, comme dans RS2 de Rocscience, résolvent les essais SSR comme des analyses individuelles et indépendantes, sans tenir compte des essais calculés précédemment. Cet article compare les résultats obtenus en utilisant les procédures SSR intégrées de RS2 à une approche à plusieurs étapes où les paramètres de résistance sont réduits manuellement, et les solutions d'essai calculées précédemment informent les modèles d'essai SSR suivants. Des modèles de pente homogènes et multi-matériaux plastiques-élastiques parfaits sont développés afin de comparer les résultats pour le FoS, le temps d'exécution et les patrons de déplacement nodal. Les modèles multi-étapes "informés" permettent une convergence des solutions d'essai et un temps de calcul global du modèle plus rapide par rapport aux analyses SSR traditionnelles, en particulier pour les maillages MEF fins et les modèles avec une tolérance de convergence plus stricte. En outre, les modèles de réduction de la résistance à plusieurs niveaux produisent des déplacements nodaux plus lisses que les solutions d'essai SSR. Les modèles de réduction de la résistance par étapes donnent des coefficients de sécurité égaux ou inférieurs à ceux de la procédure SSR intégrée pour tous les modèles de pente développés.

1 INTRODUCTION

Numerical tools such as the Finite Element Method (FEM) have become commonplace in both academia and industry for modelling the behaviour of engineered and naturally occurring slopes. One of the major questions asked by geotechnical engineers is: how close is the slope to failure? This is most often evaluated using a Factor of Safety (FoS) calculated with the Shear Strength Reduction (SSR) method which are commonly built-in to geotechnical FEM software packages. These procedures allow for a calculation of an FoS like traditional Limit Equilibrium Method (LEM) approaches such as the method of slices (e.g. Bishop, 1955; Morgernstern & Price, 1965). Griffith and Lane (1999) summarized the benefits of using FEM SSR over LEM including no assumption of shape or location of the failure surface or for slice side forces are required, compressibility data can be used to provide outputs for deformations at pre-failure states, and the FEM model can mimic progressive failure of slopes.

The SSR approach was first utilized within a FEM framework by Zienkiewicz et al. (1975). It has subsequently been described, applied and modified by authors including Naylor (1981), Donald and Giam (1988), Matsui and San (1992), Dawson et al. (1999), Griffiths and Lane (1999), Cala et al. (2004), Diederichs et al. (2007) Tschuchnigg et al (2015) and Dyson and Tolooiyan (2018), among others. SSR functions have been applied to standard geotechnical material models for soil and rock including the linear Mohr-Coulomb and the non-linear Generalized Hoek-Brown failure criteria (Hammah et al, 2005).

1.1 Definition of Failure

Different definitions have been postulated and used by various authors to determine the onset of slope failure within a FEM framework. These include exceeding a specified maximum nodal displacement, development of a continuous plastic zone within the slope, and non-convergence of the numerical model (Griffiths & Lane, 1999). When using a non-convergence definition of slope failure, a given model step must satisfy the numerical solution below a specified error tolerance within a specified number of iterations. If the model can satisfy this error tolerance, it is said to be converged. If by the maximum number of iterations, the model has not satisfied the error tolerance, the model is said to be non-converged and the slope is at a state of failure (Griffiths & Lane, 1999).

1.2 SSR using Mohr-Coulomb Strength Criterion

The Mohr-Coulomb failure criterion is a commonly used strength criterion in geotechnical engineering and is commonly applied to slope stability assessments. The FoS for a given slope is computed using Equation 1.

$$FoS = c'/c'_f = \tan\phi' / \tan\phi'_f \quad [1]$$

Where c' and ϕ' are the initially assumed effective cohesion and internal friction angle respectively and c'_f and ϕ'_f are the cohesion and internal friction angle at failure. For standard SSR analysis, c' and $\tan\phi'$ are modified simultaneously by the same factor.

Where a tensile component of strength exists, the tensile strength may be reduced simultaneously with c' and $\tan\phi'$ as shown in Equation 2.

$$FoS = t'/t'_f \quad [2]$$

Where t' is the initial tensile strength and t'_f is the tensile strength at failure.

1.2.1 Strength Reduction Factor (SRF) Notation

The term, Strength Reduction Factor (SRF), is used in combination or in the place of FoS in some literature and software documentation. This term is typically used to describe the specific factor applied to strength parameters for a given SSR trial solution. This is used to avoid confusion between the stage where failure occurs and all other trial stages where different SRF values are applied. Equations 3, 4, and 5 show this notation.

$$c'_{trial} = c'/SRF \quad [3]$$

$$\tan\phi'_{trial} = \tan\phi'/SRF \quad [4]$$

$$t'_{trial} = t'/SRF \quad [5]$$

Where c'_{trial}, ϕ'_{trial}, and t'_{trial} are the effective cohesion, internal angle of friction and tensile strength for a given SSR trial solution.

1.3 SSR Solving Procedures

Two standard procedures are used to compute the FoS when using a non-convergence definition for failure: monotonically increasing/decreasing approaches (called iSSR in this paper) or bracketing and bisecting approaches (called bSSR in this paper).

Monotonically increasing/decreasing approaches (iSSR) gradually increase or decrease the SRF applied to c' and $\tan\varphi'$ by a constant step size. For cases where the FoS is greater than 1, the SRF is monotonically increased with trial models generated at the specified step size until a non-converged model is developed. The FoS is defined based on the strength of the materials in the last converged model step. For the case where the FoS is less than 1, the SRF is monotonically decreased, with the trial models computed with decreasing strength parameters, until a converged model is developed (all prior models should not converge). The FoS is then defined using the strength parameters for the converged model step.

Dawson et al. (1999) described a bracketing and bisecting approach (bSSR) to solve for FoS. This procedure requires inputs for a lower and upper bracket SRF for the model. The lower bracket should produce a converged model and the upper bracket a non-converged model. The subsequent trial solutions use the midpoint between the two brackets to generate the next trial model. If this solution converges, this becomes the new lower bracket, if it does not converge, it becomes the new upper bracket. This procedure is repeated until the difference between the lower and upper bracket is within a specified SRF tolerance. The FoS is defined using the strength parameters applied to the last converged model.

Both solving procedures have their advantages and disadvantages. Monotonically increasing models are commonly noted by various authors to show the progressive failure of the slope as the strength degrades (Griffiths & Lane, 1999; Dyson & Tolooiyan, 2018). Whereas Dawson et al. (1999) note the FoS may be more efficiently found using a bracketed approach as it typically requires fewer trial models to be computed if reasonable assumptions of the lower and upper brackets are made.

1.4 Factors affecting SSR results

The results of a FEM SSR analysis are not unique for a given geometry with specified material parameters. Stress analysis, convergence criteria, and mesh parameters all have an impact on the model computation and results.

Stress analysis parameters include error tolerance, number of iterations, and load steps. Tolerance is the allowable error limit that a model must reach to converge. This tolerance must be achieved in a specified number of iterations. For elastic-plastic models, the loading is typically applied in incremental load steps. The number of load steps and the load step pattern (% of gravitational and external loads applied at a given load step) can be varied.

Convergence criteria use energy, force, and displacement to quantify the model error as a given model step is iterating to a solution. Convergence criteria can be defined based on one or more of the above criteria. If a more rigorous convergence criterion is utilized (i.e., one with more components) a longer computational time is expected. Additionally, model results for a given trial SRF and overall FoS can vary with convergence criteria.

The FEM mesh is one of the most well-discussed factors affecting the results of FoS, with sensitivity tests commonly reported in the literature (e.g. Tschuchnigg et al., 2015; Dyson & Tolooiyan, 2018). Generally, as the mesh becomes denser (larger number of nodes and elements) the FoS decreases, and the model run time increases.

1.5 Multi-Stage Modelling

Multi-stage FEM models are commonly used by geotechnical engineers to assess the potential behaviour of engineered and natural slopes at different stages of their life cycle. This can be applied when modelling different construction phases: for example, Wu et al. (2015) modelled the effects of embankment lifts during construction. This approach can also be used to simulate the progressive weakening of geological materials by gradually reducing material elastic and strength parameters to show the strength degradation of a slope over time. Dey and Javankhoshdel (2021) used a multi-staged approach to show the progressive failure of large-scale landslides in sensitive clay in Eastern Ontario.

There exists no literature to the authors' knowledge on using a multi-stage strength degradation approach to determine FoS for a slope stability analysis. Multi-stage approaches differ from some built-in SSR functions as they use the previous stage computations to inform the subsequent stage computations. This contrasts with some built-in SSR functions which solve each SSR trial solution independent of each other (called un-informed in this paper). For example, Rocscience's FEM software RS2 (Rocscience, 2021a) uses an informed approach for its staged models and an un-informed approach for solving the individual trials using its built-in SSR procedures (Rocscience, 2021b).

2 METHODOLOGY

A series of numerical models were developed in RS2 to compare the results of RS2's built-in SSR procedures to the results achieved using an analogous multi-stage approach. The goal of the analysis is to compare the FoS, the run times, and the nodal displacement patterns between the informed (multi-stage) and un-informed (built-in SSR) approaches.

RS2 has both monotonically increasing (iSSR) and bracketed (bSSR) solving procedures built-in to the software. Both built-in SSR procedures use an un-informed approach with each trial solved independently of the other (Rocscience, 2021b). RS2 also allows users to develop multi-stage models. The individual stages use an informed approach where the previous model step computations are used to inform the subsequent model step's computations (Rocscience, 2021b). RS2's built-in SSR procedures define failure using non-convergence. The informed multi-stage modelling will also follow this definition to compare FoS and overall model computation time. All analysis is completed using an SRF step size/tolerance of 0.01.

3 MODELS AND RESULTS

A homogeneous and a heterogeneous layered model were developed to compare and analyze the results of the built-in un-informed SSR procedures and the informed multi-stage approach. A base model was developed using the following stress analysis parameters: convergence type – Comprehensive (displacement, force, and energy criteria), error tolerance of 0.0001, maximum iterations of 2000, with 10 load steps per model stage.

All slope models were developed with the same external boundary positions and conditions (fixed along both sides and the bottom). The slopes analyzed have an angle of 45° with a height of 400 m. Slopes are gravity loaded with all material having a unit weight of 0.027 MN/m^3 and horizontal stresses defined by K = 1.

All materials used were defined using elastic-perfectly plastic Mohr-Coulomb strength parameters. Table 1 provides a summary of the material properties.

Table 1. Material elastic and strength parameters

Material Properties	Material 1	Material 2
Young's Modulus (MPa)	20000	5000
Poisson's Ratio	0.25	0.25
c' (MPa)	1	0.8
ϕ' (degrees)	35	30
T' (MPa)	0.5	0.4

Figures 1A and 1B show the homogeneous and heterogeneous slope models developed for this analysis respectively. The homogeneous slope model is entirely comprised of Material 1. The heterogeneous slope model is composed of both Material 1 and Material 2 in alternating 50m layers from the crest to the toe of the slope. Both models use 6 node triangular elements with the homogeneous slope model having 4457 elements composing the mesh whereas the heterogeneous model has 5351 elements.

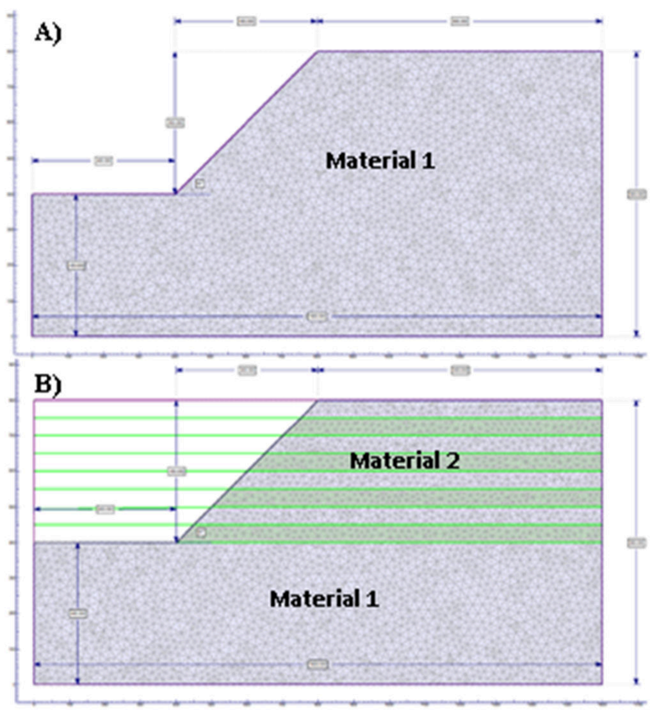

Figure 1. Slope Model Geometries A) Homogeneous B) Heterogeneous

3.1 Homogeneous Base Case

Table 2 summarizes the FoS and model run times for the homogeneous base case. Figures 2A and 2B show the failure surface of the multi-stage (M.S) and the SSR models respectively. Figure 2C shows the total nodal displacement referenced to the first SRF stage

Table 2. Homogeneous Slope Model Base Case Results

Models	FoS	Run Time (Mins)	% Time Difference (SSR-M.S)/(M.S)*100%
iSSR	1.71	21.77	441%
bSSR	1.71	7.30	82%
Multi-Stage	1.69	4.02	N/A

The FoS determined from the multi-stage approach is 0.02 less than that computed using the built-in SSR approaches. In other words, the multi-stage model failed to converge two strength reduction steps before the built-in SSR analyses. The multi-stage model resulted in less computation time than both SSR procedures with the iSSR and the bSSR taking 441% and 82% more time to compute, respectively.

There is a limited difference between the iSSR and the multi-stage model for the failure plane shown in Figures 3A & 3B which is denoted by the band of highest maximum shear values. There is also limited difference noted in the cumulative nodal displacement values referenced to stage 1 (stage with the initial strength parameters) for nodes 2 and 3 of less than 1%. For nodes 1 and 4, the difference in referenced total displacement varies between 1 to 3% for most model stages.

3.2 Heterogeneous Base Case

Table 3 summarizes the FoS and model run times for the heterogeneous base case models. Figures 3A and 3B show the failure surface of the multi-stage (M.S) and the SSR models respectively. Figure 3C shows the total nodal displacement referenced to the first SRF stage.

Table 3. Heterogeneous Slope Model Base Case Results

Models	FoS	Run Time (Mins)	% Time Difference (SSR-M.S)/(M.S)*100%
iSSR	1.48	22.43	344%
bSSR	1.48	10.42	106%
Multi-Stage	1.47	5.05	N/A

The built-in SSR and the multi-stage approaches produced FoS having a difference of 0.01 or one strength reduction step. The multi-stage model resulted in significantly faster run times with the iSSR and bSSR approaches taking 334% and 106% more time to compute respectively.

Like the homogeneous model, there is a limited difference between the failure plane shown by the maximum shear strain for the informed and un-informed procedures (Figure 3A & 3B). There are some differences between the cumulative displacement values referenced to stage 1. For Nodes 1 and 4 the percentage difference ranges from 1 to 3%. Nodes 2 and 3 range from 2 to 5% at the early strength reduction steps (~SRF<1.1) and reduce to 1 to 2% at further steps.

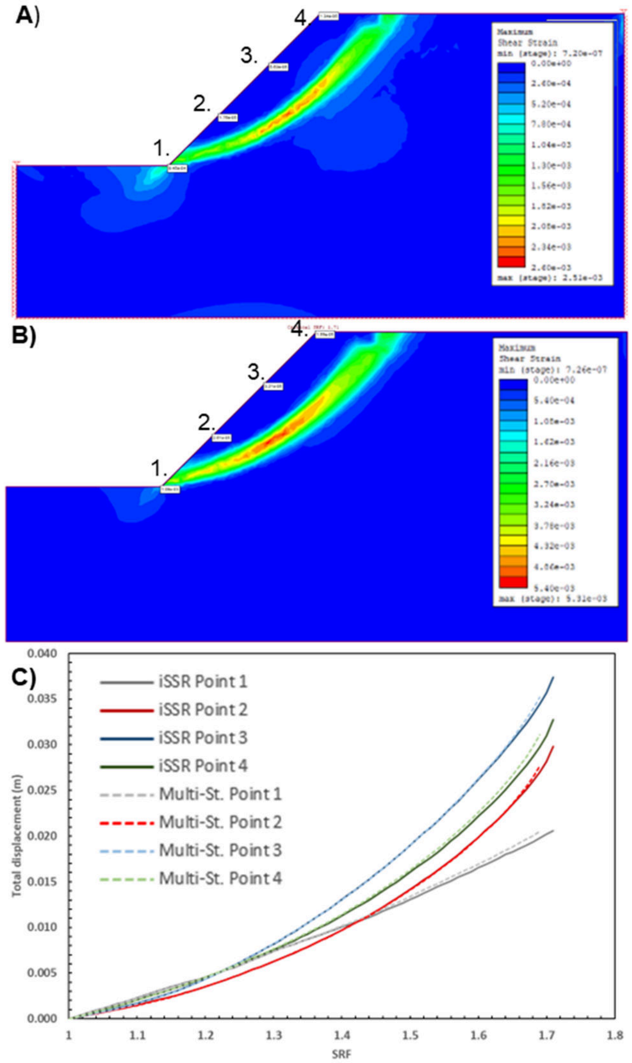

Figure 2: Homogeneous model outputs: A) Multi-stage model failure plane; B) SSR model failure plane; and C) Total cumulative nodal displacement referenced to the first stage.

3.3 Sensitivity Analysis

A sensitivity analysis of the stress analysis, convergence, and meshing parameters was completed to evaluate the effects these inputs have on the multi-stage model relative to the built-in SSR approaches. This analysis was completed using the model geometries developed for both the homogeneous and heterogeneous base cases.

Four different meshes were created to evaluate the effect of meshing parameter inputs. All meshes were composed of 6 node triangular elements equally spaced through the entire domain. Meshes are described as Coarse (least dense), Medium (base-case), Fine, and Very Fine. Table 4 summarizes the number of elements used in each case.

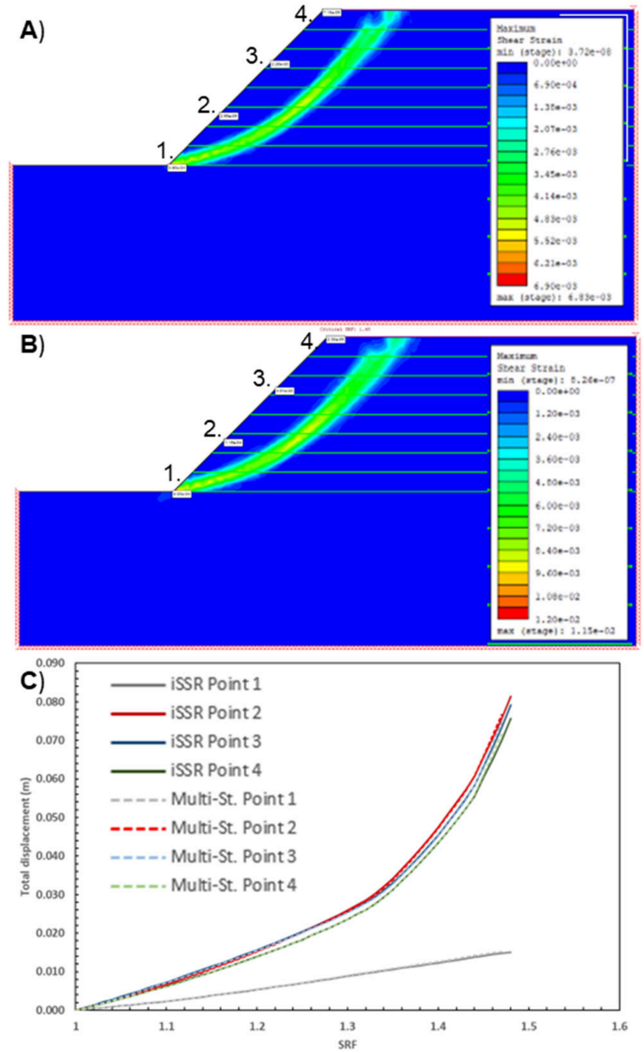

Figure 3: Heterogeneous model outputs: A) Multi-stage model failure plane; B) SSR model failure plane; and C) Total cumulative nodal displacement referenced to the first stage.

Table 4. Number of Elements in Mesh

Models	Homogeneous Model	Heterogeneous Model
Coarse	1742	1789
Medium	4457	5351
Fine	8845	9925
Very Fine	22081	19028

Models were run using all five of the convergence types available in RS2. The convergence types use different criteria to determine the model error. Three of these convergence criteria use only energy (Absolute Energy, Square Root Energy, and Enhanced Energy), one uses both force and energy (Absolute Force and Energy), and one uses force, energy, and displacement (Comprehensive – used as the base case). Detailed descriptions of the convergence criteria are available online in the RS2 documentation (Rocscience, 2021c).

Two stress analysis parameters were also evaluated: error tolerance and number of iterations. Three tolerance values were evaluated: Low (0.001), Medium (0.0005), and High (0.0001). The High tolerance is the base case value. Three values of the maximum number of iterations were also selected: Low (500), Medium (2000), and High (5000). The Medium (2000) is the base case scenario.

Tables 5 and 6 summarize the results of the sensitivity analysis for the homogeneous and the heterogeneous case respectively. The results of the base case models are repeated in the table for completeness.

A general trend is observed that with more rigorous stress analysis, convergence, and meshing parameters, the computed FoS decreases, and the model run time increases. This trend is observed specifically in the sensitivity analysis for the number of mesh elements, the number of iterations, and convergence criteria. However, the tolerance sensitivity analysis does not follow this trend. As the tolerance decreases, it does not result in longer run times for the multi-stage model. However, the pattern of decreasing FoS with increasing tolerance is observed in the multi-stage model. Further discussion of the sensitivity analysis results is presented as necessary in the following section.

4 DISCUSSION

The following section provides a discussion of outputs achieved using the multi-stage approach compared to the two built-in SSR procedures.

4.1 Difference in FoS Computed

The FoS calculated using the multi-stage approach was less than or equal to the FoS calculated using the built-in SSR procedures for all models. The informed nature of the multi-stage models results in a conservative estimate but does not drastically change the estimated value of FoS with a maximum difference of 0.02 for the homogeneous models which had a range of FoS from 1.68 to 1.72 and a maximum difference of 0.01 for the heterogeneous model which had FoS computed between 1.46 to 1.49.

This decrease in the difference in FoS computed between the multi-stage and the un-informed SSR models are expected as the model FoS approaches 1. As the FoS increases, the associated strength reduction for a given model step is less than the previous steps. This is shown in Figure 4 with a strength reduction applied to the Mohr-Coulomb cohesion parameter of Material 1 (c' = 1 MPa). Initially, as the model moves from SRF = 1 to SRF = 1.01 (one model step with an SRF step size of 0.01) the change in c' is approximately 0.01 MPa. The model's failure occurs where the change in c' is approximately 0.0035 and 0.005 MPa per step respectively for the homogeneous and heterogeneous cases. At SRF=2 the change in c' decreases to 0.0025 MPa per step. This is half that of the change in strength parameters at SRF= 1.4. So as the SRF increases there would be an expected increase in the difference in FoS, determined from the multi-stage and the built-in SSR approach results.

Table 5: Homogeneous Slope Model Sensitivity Analysis

Model	Model Type	iSSR		bSSR		Multi-Stage		% Time Diff	
		FoS	Comp. Time (min)	FoS	Comp. Time (min)	FoS	Comp. Time (min)	iSSR	bSSR
Mesh	Coarse	1.71	5.71	1.71	3.38	1.7	1.35	323	150
	Medium	1.71	21.77	1.71	7.3	1.69	4.02	442	82
	Fine	1.7	69.78	1.7	16	1.69	11.72	495	37
	Very Fine	1.69	344.76	1.69	61.33	1.68	44.15	681	39
Convergence Type	Comprehensive	1.71	21.77	1.71	7.3	1.69	4.02	442	82
	Absolute Energy	1.71	5.83	1.71	3.83	1.7	2.62	123	46
	Square Root Energy	1.72	4.68	1.72	3.83	1.71	2.25	108	70
	Enhanced Energy	1.72	4.18	1.72	5.05	1.71	2.23	87	126
	Absolute Force & Energy	1.71	20.9	1.71	7.1	1.69	4.07	414	74
Tolerance	Low (0.001)	1.71	10.19	1.71	5.5	1.7	4.48	127	23
	Medium (0.0005)	1.71	13.18	1.71	5.9	1.7	3.31	298	78
	High (0.0001)	1.71	21.77	1.71	7.3	1.69	4.02	442	82
Iterations	Low (500)	1.71	19.82	1.71	4.53	1.69	3.48	470	30
	Medium (2000)	1.71	21.77	1.71	7.3	1.69	4.02	442	82
	High (5000)	1.71	22.43	1.71	10.72	1.69	4.91	357	118

Table 6: Heterogeneous Slope Model Sensitivity Analysis

Model	Model Type	iSSR		bSSR		Multi-Stage		% Time Diff	
		FoS	Comp. Time (min)	FoS	Comp. Time (min)	FoS	Comp. Time (min)	iSSR	bSSR
Mesh	Coarse	1.49	4.78	1.49	2.01	1.49	1.32	262	52
	Medium	1.48	22.43	1.48	9.7	1.47	5.05	344	92
	Fine	1.46	55.28	1.46	21.83	1.46	13.1	322	67
	Very Fine	1.46	159.13	1.46	69.48	1.46	32.53	389	114
Convergence Type	Comprehensive	1.48	22.43	1.48	9.7	1.47	5.05	344	92
	Absolute Energy	1.48	5.47	1.48	9.78	1.48	2.78	97	252
	Square Root Energy	1.48	4.57	1.48	4.17	1.48	2.3	99	81
	Enhanced Energy	1.49	3.95	1.49	3.28	1.48	2.45	61	34
	Absolute Force & Energy	1.48	21.8	1.48	9.97	1.47	5.33	309	87
Tolerance	Low (0.001)	1.48	11.08	1.48	11.33	1.48	5.6	98	102
	Medium (0.0005)	1.48	13.63	1.48	7.31	1.48	6.38	114	15
	High (0.0001)	1.48	22.43	1.48	10.42	1.47	5.05	344	106
Iterations	Low (500)	1.48	20.26	1.48	6.32	1.47	4.47	353	41
	Medium (2000)	1.48	22.43	1.48	10.42	1.47	5.05	344	106
	High (5000)	1.48	22.95	1.48	28.13	1.47	6.25	267	350

The model developed for this paper had FoS values that are within or slightly higher than the design acceptability criteria recommended by Wesseloo and Read (2009) for open pits slopes with a high consequence of failure (FoS of between 1.3 and 1.5). The heterogeneous models with FoS typically around 1.5 had no cases where the FoS computed had a difference greater than 0.01 between the multi-stage and built-in SSR function results. So, within the design acceptability criteria for open pit slopes, a change in the strength of 0.4 – 0.5% is the difference between the multi-stage and the SSR approaches. Other factors such as those evaluated in the sensitivity analysis yield similar or larger changes in FoS than using an informed or un-informed procedure for calculating FoS.

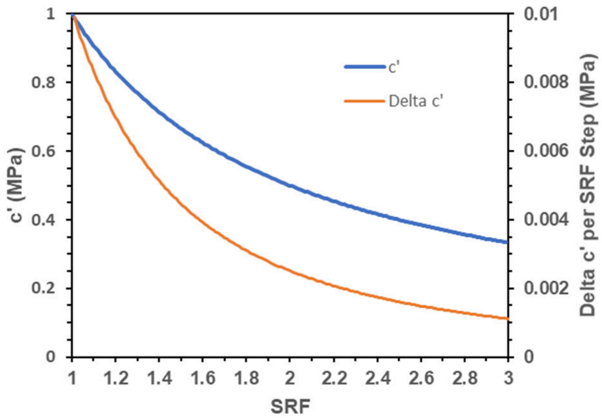

Figure 4. Change in c' ($c_{initial}$ = 1 MPa) as a function of SRF.

4.2 Run Time and Numerical Efficiency

The informed multi-stage models were faster than both built-in SSR functions for all models developed. For the homogeneous model, the iSSR was between 87% (convergence type – Enhanced Energy) to 681% (mesh – Very Coarse) slower and bSSR was between 23% (tolerance – Low) to 150% (mesh – Coarse). For the heterogeneous models the iSSR was between 61% (convergence – Enhanced Energy) and 252% (mesh – Very Fine) and bSSR was between 15% (tolerance – Medium) and 252% (convergence – Square Root Energy).

The informed multi-stage approach results in faster run times as the number of iterations required to converge both a single load step and the entire model stage is lower than the un-informed SSR procedures. Figure 5 shows the number of iterations for two stages from the homogeneous base case as an example. The multi-stage model was able to converge the first 9 out of the 10 total load steps up to SRF=1.68 in a single iteration. Only in the two model stages before non-convergence did multiple load steps require more than a single iteration to converge. Compared to the built-in SSR approaches, the number of iterations required is generally between 50 - 150 iterations for all load steps at every SRF trial model.

Dawson et al. (1999) noted that the bSSR approaches are more computationally efficient than iSSR approaches. This is seen to be true in most of the models run in this analysis, but not all. If a poor upper bracket is used to bound the solution, then the model will have to calculate multiple non-converging stages, which can require multiple load steps computing the maximum number of iterations set. The run time for the last converged homogeneous base case stage is 24 seconds compared to 57 seconds to run the closest non-converged stage. The run-time for non-converge stages increases as the SRF increases away from the model FoS.

Ultimately the more iterations required, the longer the model takes to compute. The multi-stage model took between 2 to 3 seconds to converge a given model step whereas the iSSR model took between 14 seconds for early strength reduction steps and up to 24 for the trial solution before failure. While the length of these run times is small, the significance increases with more complex models.

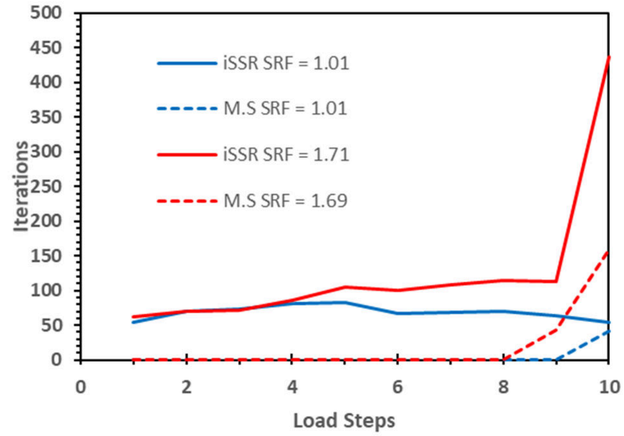

Figure 5. Number of iterations required to converge homogeneous base case model vs. load step.

4.3 Modelling Progressive Failure of Landslides

Griffith and Lane (1999) and Dyson & Tolooiyan, (2018) note that one of the major advantages of using an increasing SSR (iSSR) approach is the number of trial solutions developed better allows for visualization of the progressive failure of slopes. Bracketed approaches provide fewer trial solutions before failure limiting the ability to visualize the progressive nature of the failure and trends in the results. However, the un-informed nature of both traditional SSR approaches results in stage-specific displacements spiking up and down as opposed to a gradual increasing pattern as one may expect. This is especially the case for small SRF step sizes such as an SRF step of 0.01 used in this paper.

Figure 6 shows the stage-specific total displacement for the homogeneous base case for both the informed multi-stage model and the un-informed iSSR approach. Overall, the multi-stage model produces a gradually increasing curve for the stage-specific total displacements compared to the un-informed iSSR approach. This is best seen for nodes 2 and 3. The un-informed stage-specific nodal displacement although generally following an increasing trend, the curve is not smooth like the informed model results and spikes above and below the displacement curves developed using the informed multi-stage model. This effect is not well observed on the total referenced nodal displacement curve (seen in Figures 2C and 3C) due to the small magnitude of the stage-specific values compared to the overall total nodal displacement. However, it does result in small percentage differences (~1-3%) between the informed and un-informed models at different model steps.

It would be expected that with each strength reduction step, the nodal displacement in each step would continue to increase as is seen in the results of the multi-stage model. For this reason, an informed multi-stage model may be better at capturing the progressive failure of engineered and naturally occurring results than un-informed iSSR approaches and be better for calibrating instrumentation monitoring data to the FEM model.

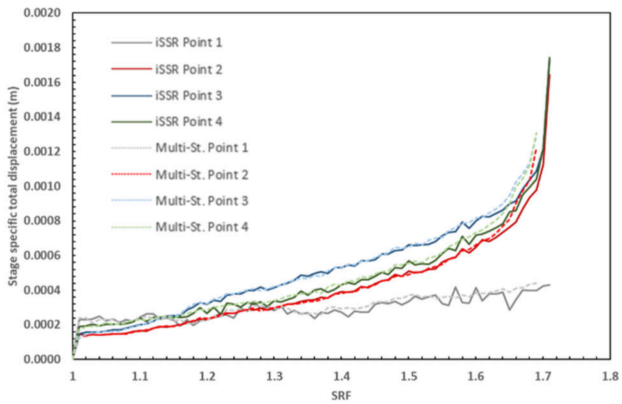

Figure 6. Stage-specific total displacement for homogeneous base case model for informed multi-stage and un-informed iSSR models.

4.4 Serviceability Limit State Analysis

As noted in Section 4.3, one of the major advantages of using an increasing approach is it allows for the model to show the progressive nature of the developing slope failure as opposed to bracketed approaches where there are fewer trial models before failure. The FoS computed using SSR procedures is for the ultimate failure of the slope. However, other criteria such as serviceability limit states use displacement as a metric for failure. The limited pre-failure trials generated during bracketed approaches potentially provide insufficient data points to determine an FoS associated with the serviceability of a given slope as a gap may exist where the allowable displacement value is surpassed.

Using the heterogeneous slope model base case as an example, the ultimate FoS occurs for both the multi-stage and built-in SSR approaches at 1.47 and 1.48 respectively. However, for the trial solutions with an SRF of 1.3 or greater, there is a distinct acceleration in the total nodal displacement compared to the previous model steps. This point where the nodal displacements positively inflect may be more relevant to the slope design or monitoring in certain scenarios than the ultimate value.

The overall run-time for iSSR models is significantly longer than using a bracketed approach. Using an informed approach for solving slope stability problems provides both the benefit of being faster than traditional un-informed bracketed approaches as well as provides a fuller picture of the expected slope displacement pattern. This could provide the additional ability for engineers to determine a serviceability FoS using a displacement criterion in addition to a single ultimate failure value.

5 CONCLUSIONS

This paper presents a comparative analysis of standard built-in SSR approaches that are commonly used in geotechnical engineering of slopes to determine an FoS and a multi-staged model using Rocscience's RS2. The multi-stage analysis uses an informed approach where computations from previous model steps are used to inform a subsequent model stage's computation. This differs from the built-in SSR procedures where each trial model is calculated independently of all other trial solutions.

Two homogeneous and heterogeneous slope models were developed with elastic-perfectly plastic Mohr-Coulomb materials. An analysis was completed on a base case with standard stress analysis, convergence, and meshing parameters utilized. A sensitivity analysis was completed by varying the base case analysis parameter to evaluate the relative effect it has on the solving of the slope models using the two approaches. While the slope is reflective of an idealized pit slope geometry, the results are equally applicable to large natural slopes.

The following major observations are made based on the results of the base case and the sensitivity analysis:

1. The multi-stage approach resulted in FoS less than or equal to the FoS computed using the built-in SSR approaches. However, the difference between the FoS computed using the multi-stage approach and the un-informed SSR approaches was a maximum of 0.02 or two strength reduction steps less than the SSR analysis.
2. The multi-stage models resulted in faster run times for all base case and sensitivity analysis models compared to the increasing (iSSR) and bracketed (bSSR) approaches.
3. Multi-stage models produce smoother nodal displacement curves than the iSSR approach for high tolerance cases.

As the informed approach produces similar FoS values whilst being more computationally efficient, this solving procedure has the potential to be applied to computationally intensive models (e.g. dense meshes, complex geometries) or in probabilistic analyses such as Monte Carlo simulations where hundreds or thousands of models are required to be run. By using an informed approach modellers can save significant time while having minimal changes in results compared to standard SSR procedures.

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