How to obtain representative friction angle of coarse granular materials with not large enough specimens

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ABSTRACT
Direct shear test is a commonly used method to determine the shear strength of geomaterials. The authors' recent studies showed that the minimum required ratio of 10 between the specimen width (W) to the maximum particle size (d_{max}) of granular material stipulated by ASTM D3080/D3080M-11 is not large enough to eliminate the specimen size effect (SSE). Using this minimum required ratio can result in overestimation of the friction angle and leads to non-conservative structure design. The authors' studies further showed that a minimum required ratio of 60 is appropriate to eliminate the SSE. This is however problematic for coarse granular materials like gravel, rockfill and waste rocks to prepare specimens having a W/d_{max} ratio of 60 even with large direct shear box of 30 cm. In this article, an equation is presented to describe the relationship between the normalized friction angle and W/d_{max} ratio. Direct shear tests and pile tests are performed to validate the proposed equation. By using this equation along with the friction angles obtained by direct shear tests on specimens having W/d_{max} ratios smaller than 60, the friction angles corresponding to the specimens having W/d_{max} ratio of 60 can be obtained that are exempt from SSE.

RÉSUMÉ
La méthode de cisaillement direct est couramment utilisée pour déterminer la résistance au cisaillement des géomatériaux. Les études récentes menées par les auteurs ont montré que la valeur de 10 pour le ratio minimum requis entre la largeur d'une éprouvette (W) et la taille maximale des particules du matériau granulaire (d_{max}) stipulée par la norme D3080/D3080M-11 de l'ASTM n'est pas suffisamment grande pour éliminer l'effet de taille de l'éprouvette (ETE). L'utilisation de cette valeur minimale requise peut mener à une surestimation de l'angle de frottement et conduit à une conception de structures non conservatrice. Les études suggèrent qu'une valeur de 60 est nécessaire pour éliminer le ETE. Cette exigence est problématique pour les matériaux granulaires grossiers comme le gravier, le remblai rocheux et les roches stériles afin d'obtenir des éprouvettes ayant une valeur de 60 pour le ratio de W/d_{max}, même avec une grande boîte de cisaillement directe de 30 cm. Dans cet article, une équation a été présentée pour décrire la relation entre l'angle de frottement normalisé et le ratio de W/d_{max}. Des essais de cisaillement direct et des essais de pile sont effectués pour valider l'équation proposée. En utilisant cette équation, l'angle de frottement correspondant à une éprouvette ayant un ratio de 60 peut être obtenu à partir des angles de frottement obtenus par des essais de cisaillement direct sur des éprouvette ayant un ratio de de W/d_{max} plus petit que 60.

1 INTRODUCTION

Direct shear test is a popular and commonly used method to measure the shear strength parameters of geomaterials. Despite some limitations, it remains largely used due to its simplicity and cost effectiveness. It is also the prevalent method to determine the behavior of rock joints subjected to shear loading (Sow et al. 2016; Bahaaddini 2017; Zhang et al. 2019; Morad et al. 2020) or material interfaces (Choudhary and Krishna 2016; Punetha et al. 2017; Afzali-Nejad et al. 2017, 2018; Xu et al. 2019).

Direct shear test method imposes a sliding plane. This aspect represents an important drawback of the testing method. When the specimen size is too small, the influence of individual particles along the shear plane on the shear strength can be amplified and the test results can become non-representative of the shear strength in field conditions. The variation of shear strength with specimen size is known as specimen size effect (SSE) (Parsons 1936; Dadkhah et al. 2010; Mirzaeifar et al. 2013; Ziaie Moayed et al. 2017; Deiminiat et al. 2020; Zahran and Naggar 2020; MotahariTabari and Shooshpasha 2021; Deiminiat et al. 2022).

Until now, several norms (BS 1377, Eurocode 7, ASTM D3080) have been proposed and widely implemented to specify the specimen size and maximum particle size (d_{max}) of tested materials. ASTM requires a ratio of at least 10 between the specimen width (W) and d_{max}. For granular material having a d_{max} value smaller than 1 mm, one can readily obtain specimens having W/d_{max} ratio equal to or larger than 60 with standard direct shear apparatus and a shear box of 60 mm. For coarse granular materials like gravel, rockfill and waste rocks, the minimum required ratio of 10 is almost always used by people because associated d_{max} values are very large. Special large direct shear apparatus must be employed to reach the minimum required W/d_{max} ratio of 10. However, a literature review given by Deiminiat et al. (2020) showed that this minimum required W/d_{max} ratio of 10 is arguably not large enough to systematically avoid SSE. This was further confirmed by an experimental study of Deiminiat et al. (2022).

It should be noted that Deiminiat et al. (2020, 2022) are not the first ones who investigated the SSE of direct shear tests. Rather, several publications on this aspect have been reported over the years (e.g., Parsons 1936; Rathee 1981; Jewell and Wroth 1987; DeJong et al. 2003; Hight and Leroueil 2003; Cerato and Lutenegger 2006; Wang et al. 2007; Wu et al. 2008; Wang and Gutierrez 2010; Dadkhah et al. 2010; Mirzaeifar et al. 2013; Ziaie Moayed et al. 2017; Zahran and Naggar 2020). However, only a few of them (e.g., Palmeira and Milligan 1989; Cerato and Lutenegger 2006) studied the variation of shear strength as a function of specimen size ratios while keeping all other influencing factors (such as material, d_{max} value, density, moisture, etc.) constant. These studies along with the experimental work of Deiminiat et al. (2022) showed that the minimum required W/d_{max} ratio of 10 stipulated by ASTM D3080/D3080M-11 is too small to avoid any SSE on shear test results. Deiminiat et al. (2022) further indicated that the minimum required W/d_{max} ratio of 60 is appropriate to eliminate the SSE of direct shear tests. Making specimens having this minimum required ratio is not a problem for fine particle materials with d_{max} smaller than 1 mm even with standard shear box of 60 mm in width. For coarse granular materials such as gravel, rockfill and waste rocks, which usually have large d_{max} values, preparing a specimen with the minimum required W/d_{max} ratio of 60 can become complicated, if not impossible.

In this paper, an equation is presented to describe normalized friction angle as a function of W/d_{max} ratio. The equation is derived from applying best fitting technique to the normalized friction angles of the existing data. By using this equation, the friction angle corresponding to specimens having W/d_{max} ratio of 60 can be obtained from friction angles obtained by direct shear tests on specimens having W/d_{max} ratios smaller than 60.

2 PROPOSED EQUATION RELATING NORMALIZED FRICTION ANGLE AND W/d_{max} RATIO

Table 1 is a reproduction of a table published in Deiminiat et al (2022). The table includes the authors' experimental data and a part of selected experimental results obtained by using an adequate methodology, in which the variation of friction angle is only due to the variation of specimen size. Another criterion for the section of experimental data is the availability of experimental results obtained on large enough specimens having a W/d_{max} ratio of 60. For each material with a given d_{max}, particle shape, density (or void ratio), moisture content, ratio and under the same range of normal stresses, the friction angle ($\phi_{W/d_{max}}$) obtained on specimens of any W/d_{max} ratios can then be normalized by the friction angle (ϕ_{60}) obtained on specimens having a W/d_{max} ratio of 60. A relationship can be established between the normalized friction angle ($\phi_{W/d_{max}}/\phi_{60}$) and W/d_{max} ratio.

Table 1. Normalized friction angles of experimental results available in the literature (a reproduction of Table 12 of Deiminiat et al. 2022)

Material	W/d_{max}	$\phi_{W/d_{max}}$ (°)	$\frac{\phi_{W/d_{max}}}{\phi_{60}}$	Reference
Sand	50	50.1	1.014	Palmeira and Milligan (1989)
	833	49.4	1	
Gravel, D_r = 25%	20	36.5	1.074	Cerato and Lutenegger (2006)
	61	34.0	1	
Gravel, D_r = 85%	20	43.0	1.024	
	61	42.0	1	
Gravel, D_r = 55%	20	41.0	1.020	
	61	40.2	1	
WR1, d_{max} = 0.85 mm	45	37.1	1.005	Deiminiat et al. (2022)
	353	36.9	1	
WR1, d_{max} = 1.19 mm	32	38.0	1.013	
	50	37.9	1.011	
	252	37.5	1	
WR1, d_{max} = 1.4 mm	27	38.7	1.027	
	43	38.0	1.008	
	214	37.7	1	
WR1, d_{max} = 2.36 mm	16	40.9	1.082	
	25	39.1	1.034	
	127	37.8	1	
WR1, d_{max} = 3.36 mm	11	42.1	1.088	
	18	40.2	1.039	
	89	38.7	1	
WR1, d_{max} = 5 mm	12	41.4	1.048	
	60	39.5	1	
WR2, d_{max} = 0.85 mm	45	35.3	1.009	
	71	35.2	1.006	
	353	35.0	1	
WR2, d_{max} = 1.19 mm	32	36.2	1.006	
	50	36.1	1.002	
	252	36.0	1	
WR2, d_{max} = 1.4 mm	27	37.2	1.028	
	43	36.4	1.006	
	214	36.2	1	
WR2, d_{max} = 2.36 mm	16	38.2	1.030	
	25	37.3	1.005	
	127	37.1	1	
WR2, d_{max} = 3.36 mm	11	40.5	1.083	
	18	39.3	1.051	
	89	37.4	1	
WR2, d_{max} = 5 mm	12	40.1	1.044	
	60	38.4	1	

Figure 1 shows the variation of normalized friction angle ($\phi_{W/d_{max}}/\phi_{60}$) as a function of W/d_{max}. An application of curve-fitting technique leads to an exponential function as follows:

$$\phi_{W/d_{max}}/\phi_{60} = a\exp[1/W/d_{max}^{b}], \text{ for } 60 \geq W/d_{max} \geq 10 \quad [1a]$$

$$\phi_{W/d_{max}}/\phi_{60} = 1, \text{ for } W/d_{max} \geq 60 \quad [1b]$$

with a = 0.98 and b = 0.92

Even though Equation 1 was obtained by applying the curve-fitting technique on a limited number of experimental results, it can be considered as a general solution because the experimental data used in the curve-fitting process are of different sources. The equation can then be used to predict the value of ϕ_{60} of coarse granular materials once a $\phi_{W/d_{max}}$ is obtained by direct shear tests on not large enough specimens.

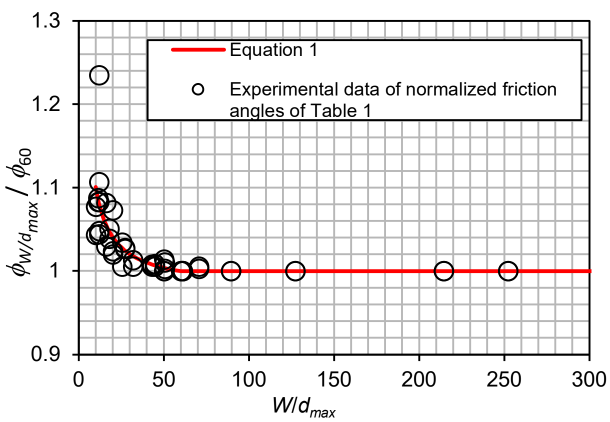

Figure 1. The normalized friction angles versus W/d_{max}

3 VALIDATION TEST OF THE PROPOSED EQUATION

3.1 Direct shear tests

To test the validity of the proposed equation on granular materials with $d_{max} \geq 5$ mm, direct shear tests were first performed using a large shear box of 300 mm × 300 mm × 180 mm. The test specimens have thus W/d_{max} ratio smaller than 60. Equation 1 must be used to predict the friction angle corresponding to a W/d_{max} ratio of 60. Secondly, pile tests were performed on the same granular materials. The measured repose angles can then be used to test the validity of the predicted friction angles because the repose angle of a granular material is well-known to be equal to the internal friction angle of the granular material determined by direct shear test at its loosest state (Miura et al. 1997; Ghazavi et al. 2008; Fu et al. 2020; Zheng et al. 2021).

Figure 2 shows the grain size distribution of the three tested materials, which were made from a waste rock by applying the scalping technique and excluding the over-sized particles. The obtained materials M1, M2, and M3 have d_{max} values of 9.5, 19 and 25 mm, respectively. It should be noted that d_{max} values are used here as an identification of the material because the scope of this study is to analyze the SSE, not the influence of d_{max} on the shear strength of material.

The direct shear tests were carried out with the three granular materials prepared at the loosest state. Table 2 presents characteristics of the prepared specimens. The specific gravity (G_s) and the maximum void ratio (e_{max}) of the specimens are presented.

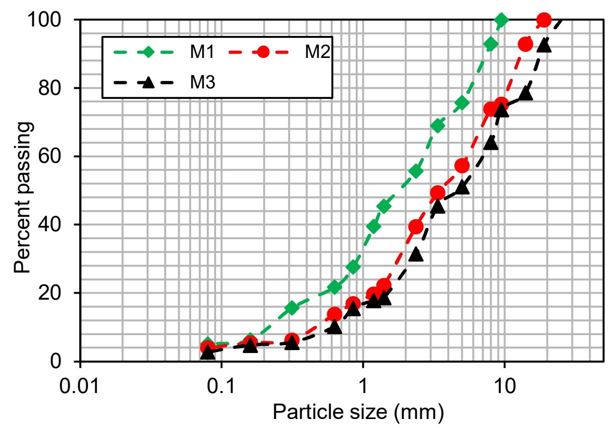

Figure 2. Grain size distribution curves of the materials

Table 2. Specimens prepared for direct shear tests

Material	G_s (ASTM C127-15)	e_{max}	Large shear box	
			W/d_{max}	T/d_{max}
M1, d_{max} = 9.5 mm	2.60	0.79	32	19
M2, d_{max} = 19 mm	2.53	0.74	16	9
M3, d_{max} = 25 mm	2.56	0.69	12	7

For each material, the friction angle was measured three times. The applied normal stresses were 50, 100, and 150 kPa. The shear load was applied by a shear rate of 0.025 mm/s. Table 3 shows the three friction angles obtained by direct shear tests for each material. The average values and the predicted ϕ_{60} are also presented in the table. As an example of calculations for the specimen of M3 with the d_{max} value of 25 mm and the W/d_{max} ratio equal to 12, the measured $\phi_{W/d_{max}}$ is 45.2°. Applying Equation 1 leads to:

$$\phi_{60} = 45.2° /0.98^{*}\exp [1/12^{\wedge}0.92] = 41.8°$$

Table 3. The friction angles $\phi_{W/d_{max}}$ measured by direct shear tests

Material	W/d_{max}	$\phi_{W/d_{max}}$ (°)	Avg. $\phi_{W/d_{max}}$ (°)	ϕ_{60} (°)
M1, d_{max} = 9.5 mm	32	40.8 41.2 41.5	41.2	40.5

M2, d_{max} = 19 mm	16	43.4 43.7 44.5	43.9	41.6
M3, d_{max} = 25 mm	12	44.9 45.1 45.5	45.2	41.8

3.2 Pile tests

To test if the predicted ϕ_{60} values given in Table 3 are valid, pile tests were performed on each of the three tested granular materials. The pile tests were performed by keeping a funnel completely close to the top of the growing heap, as shown in Figure 3. On the figure, h_p is the height of the pile (mm) and d_p is the bottom diameter of the pile (mm). The repose angle ϕ_p can be obtained by:

$$\phi_p = \tan^{-1}(2h_p / d_p) \qquad [2]$$

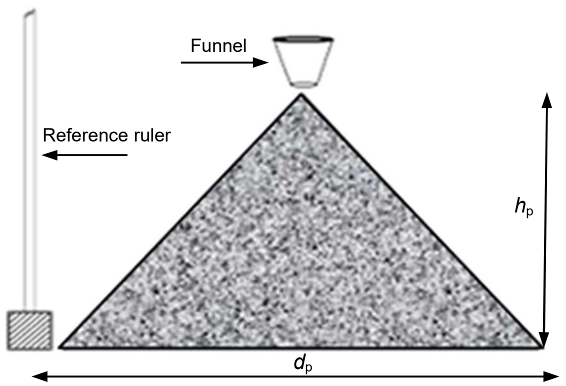

Figure 3. Schematic view of pile test setup in the laboratory

Figure 4 shows the variation of measured repose angles as a function of pile height h_p, normalized by d_{max} value for materials M1 and M3. The results tend to indicate that there is no size effect in the measurement of repose angle.

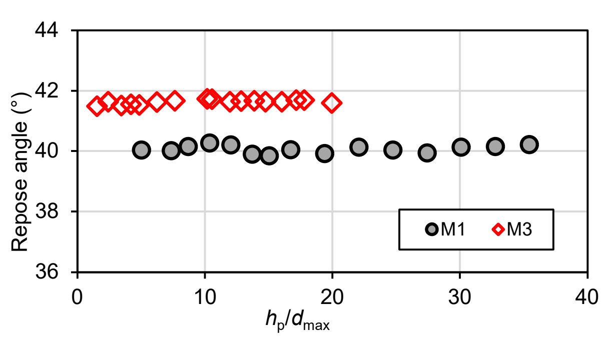

Figure 4. Variation of the ϕ_p with h_p/d_{max} for M1 and M3

Table 4 shows the repose angles and the corresponding average values of the three tested granular materials. Despite the scope of this study is not on the influence of d_{max} on the shear strength, it is interesting to note that the repose angle increases as the d_{max} value increases. This corresponds well to what one usually observes in field with rockfill dam or waste rock piles. A waste rock pile having larger d_{max} values usually exhibits a larger repose angle than a structure made of sand with smaller d_{max} value.

Table 4. The ϕ_p and their average values obtained by the pile tests for the three materials

Material	h_p (mm)	$d_p/2$ (mm)	ϕ_p (°)	Avg. ϕ_p (°)
M1, d_{max} = 9.5 mm	176 175 172	209 210 205	40.1 39.8 40.0	40.0
M2, d_{max} = 19 mm	172 177 180	200 201 203	41.0 41.4 41.6	41.3
M3, d_{max} = 25 mm	180 177 176	201 199 198	41.8 41.7 41.6	41.7

3.3 Results analysis

Table 5 shows comparisons between the friction angles (ϕ_{60}) predicted by applying Equation 1 and the measured repose angles (ϕ_p). The good agreements between the predicted and measured friction angle indicate that Equation 1 can be used to determine the ϕ_{60} value through direct shear tests on not large enough specimen.

Table 5. The ϕ_{60} values predicted by applying Equation 1 to the measured $\phi_{W/d_{max}}$ of not large enough specimens and the measured ϕ_p values

Material	Measured ϕ_p (°)	Predicted ϕ_{60} (°)
M1, d_{max} = 9.5 mm	40.0	40.5
M2, d_{max} = 19 mm	41.3	41.6
M3, d_{max} = 25 mm	41.7	41.8

4 DISCUSSION AND CONCLUSIONS

In this work, an equation was developed to describe the friction angle as a function of the W/d_{max} ratio. The validity of this equation has been tested using direct shear test and pile test outputs. However, it should be noted that the experimental data used for validation are limited to the d_{max} value of 25 mm. In addition, only one type of waste rock was used to prepare the materials. More experimental works on different materials with larger particle sizes having different properties are necessary to see if the equation still remains valid.

The pile test results indicate that the measured repose angles are insensitive to the pile size. These results tend to indicate that the repose angle of large waste rock piles can be determined by performing small pile tests. More experimental tests are required to see whether this conclusion is universally valid.

The results of this study further showed that the friction angle increases as the d_{max} value increases. This trend is not in the agreement with that reported by the previous studies for angular particle materials (Varadarajan et al. 2003, 2006; Abbas 2011; Honkanadavar et al. 2014; Dorador et al. 2017, 2020; Deiminiat et al. 2020). This observation is probably related to the low normal stresses used in this study compared to the large normal stresses were applied to the test specimens in the previous studies. The decrease in friction angle with increase in d_{max} values has also been explained by the size effect of rock strength (Baecher and Einstein 1981; Li et al. 1999, 2001; Aubertin et al. 2000; Sheng-Qi et al. 2005; Li et al. 2007; Ovalle et al. 2014), but it is not clear if the SSE was considered in the results reported by these studies.

Despite the above mentioned, it can be concluded that the proposed equation has been validated, at least partly. It can be used to determine the friction angle of coarse granular materials exempt of SSE by direct shear tests on not large enough specimens.

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